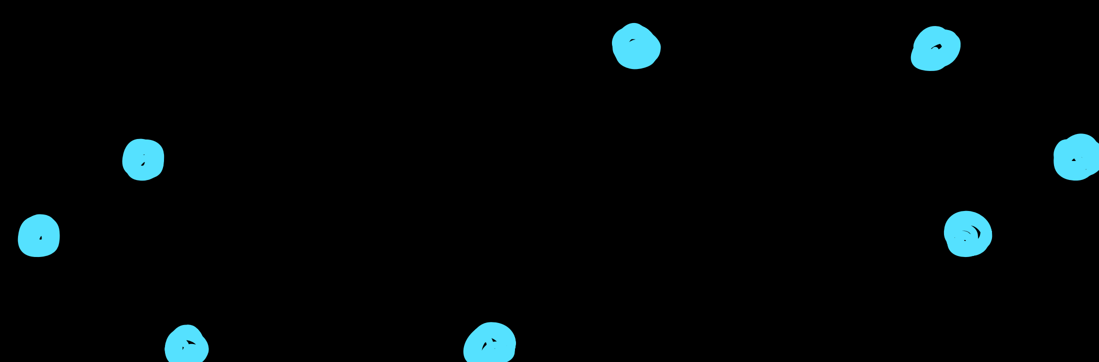
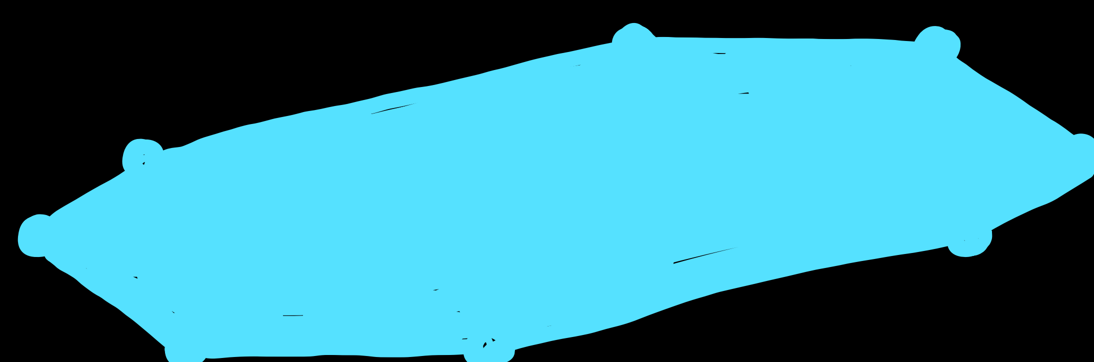
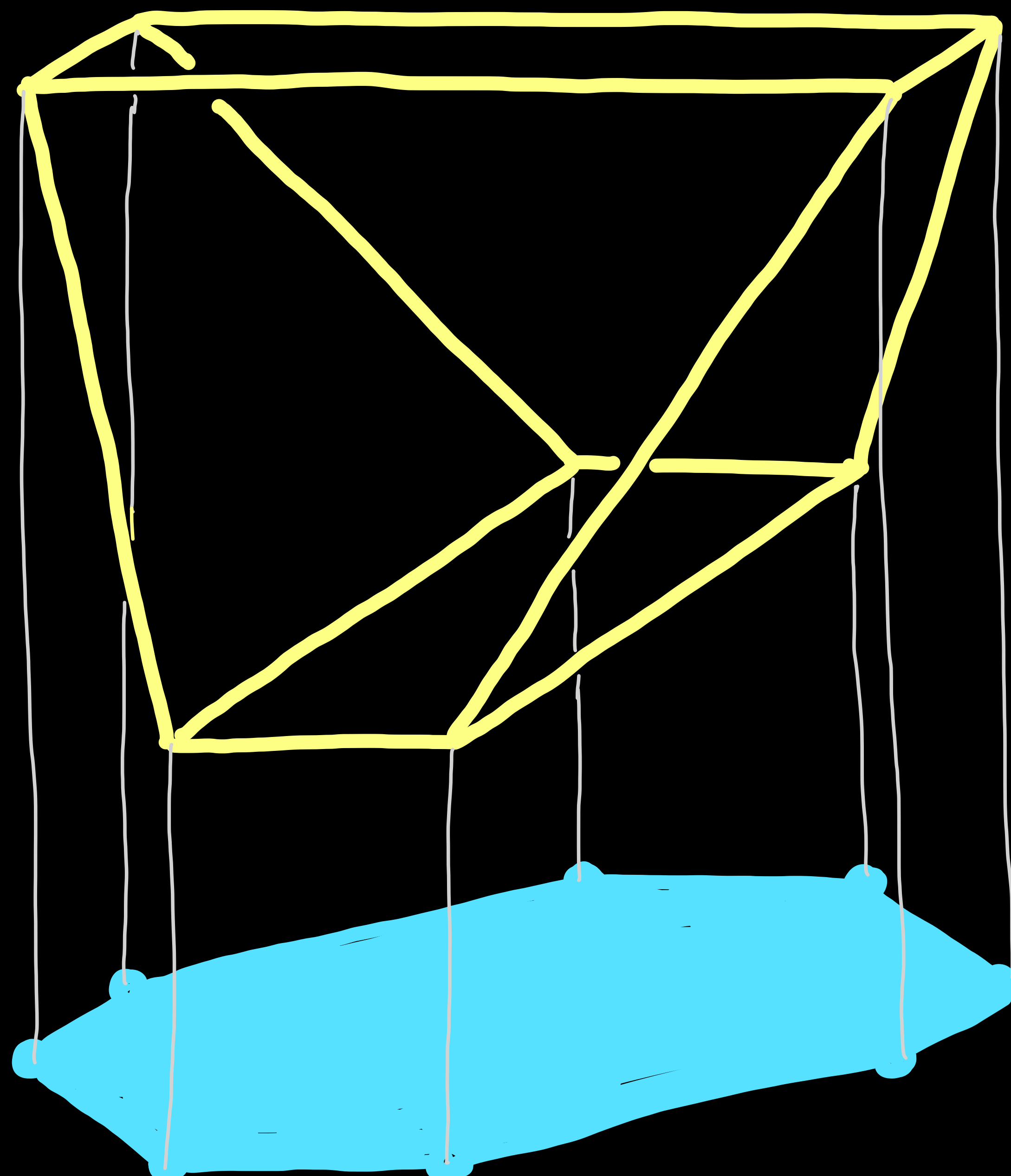
$xc(\mathrm{CORR}(n)) \geq 1.5^n$

VOLKER KAIBEL
(OVGU MAGDEBURG)

BASED ON JOINT WORK WITH
STEFAN WELTGE
(TU MÜNCHEN)

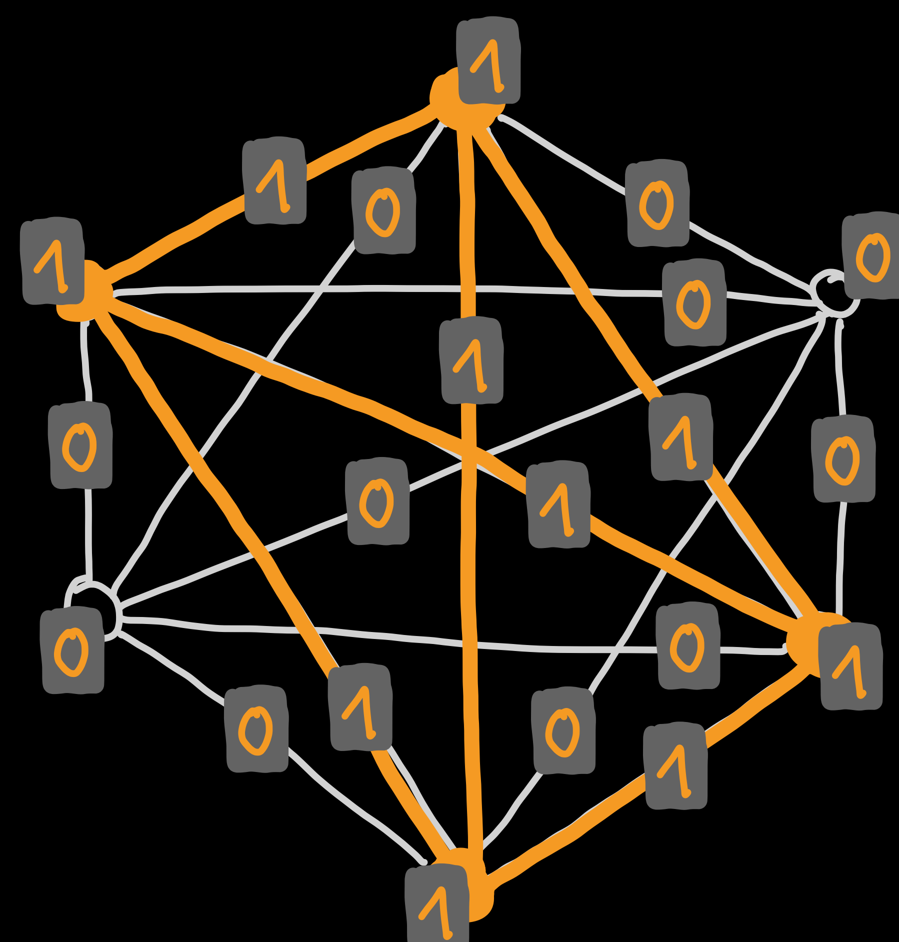






THE CORRELATION POLYTOPE

$$\mathrm{CORR}(n) := \mathrm{conv}\left\{ \left(\chi(A), \chi(E(A)) \right) : A \subseteq [n] \right\}$$



THEOREM [K. & WELTGE 2013]

$$xc(\mathrm{CORR}(n)) \geq 1.5^n$$

THEOREM [K. & WELTGE 2013]

$$\mathrm{XC}\left(\mathrm{CORR}(n)\right) \geq 1.5^n$$

EXTENSION COMPLEXITY

=

SMALLEST NUMBER OF FACETS
OF A POLYTOPE
WHOSE PROJECTION IS CORR(n)

FIORINI, MASSAR, POKUTTA, DE WOLF, TIWARY 2012

$$\exists \varepsilon > 0 : \quad xc(\mathrm{CORR}(n)) \geq (1+\varepsilon)^n$$

$$\left(\Rightarrow \; xc(\mathrm{TSP}(n)) \geq 2^{\Omega(\sqrt{n})}, \ldots \right)$$

FIORINI, MASSAR, POKUTTA, DE WOLF, TIWARY 2012

$$\exists \varepsilon > 0 : \quad xc(\mathrm{CORR}(n)) \geq (1+\varepsilon)^n$$

$$\left(\Rightarrow \; xc(\mathrm{TSP}(n)) \geq 2^{\Omega(\sqrt{n})}, \ldots \right)$$

BRAUN & POKUTTA 2013

$$xc(\mathrm{CORR}(n)) \geq 1.24^n$$

EXAMPLE 1 : PERMUTAHEDRON

$$\mathrm{PERM}(n) := \mathrm{conv}\left\{ (\sigma(1), \dots, \sigma(n)) : \sigma : [n] \to [n] \text{ BIJECTION} \right\}$$

EXAMPLE 1 : PERMUTAHEDRON

$$\mathrm{PERM}(n) := \mathrm{conv}\left\{ (\sigma(1), \ldots, \sigma(n)) : \sigma : [n] \to [n] \text{ BIJECTION} \right\}$$

$$\overset{[\text{WOLSEY } 86]}{=} \left\{ \left(\sum_{j=1}^{n} y_{ij} \right)_{i=1}^{n} : 0 \le y_{ij} \underset{(i \neq j)}{=} 1 - y_{ji} \le 1 \right\}$$

EXAMPLE 1 : PERMUTAHEDRON

$$\mathrm{PERM}(n) := \mathrm{conv}\left\{ (\sigma(1), \dots, \sigma(n)) : \sigma : [n] \to [n] \text{ BIJECTION} \right\}$$

$$\overset{\text{[WOLSEY 86]}}{=} \left\{ \left(\sum_{j=1}^{n} y_{ij} \right)_{i=1}^{n} : 0 \le y_{ij} \underset{(i \neq j)}{=} 1 - y_{ji} \le 1 \right\}$$

$$\overset{\substack{\text{[BIRKHOFF]} \\ \text{[V. NEUMANN]}}}{=} \left\{ \left(\sum_{j=1}^{n} j \cdot z_{ij} \right)_{i=1}^{n} : z_{ij} \ge 0, \sum_{j=1}^{n} z_{ij} = 1 \ \forall i, \sum_{i=1}^{n} z_{ij} = 1 \ \forall j \right\}$$

EXAMPLE 1: PERMUTAHEDRON

$$\mathrm{PERM}(n) := \mathrm{conv}\left\{ (\sigma(1), \dots, \sigma(n)) : \sigma : [n] \to [n] \text{ BIJECTION} \right\}$$

$$[\text{WOLSEY } 86] = \left\{ \left(\sum_{j=1}^{n} y_{ij} \right)_{i=1}^{n} : 0 \le y_{ij} \underset{(i \neq j)}{=} 1 - y_{ji} \le 1 \right\}$$

$$\begin{array}{l}[\text{BIRKHOFF}] \\ [\text{V. NEUMANN}]\end{array} = \left\{ \left(\sum_{j=1}^{n} j \cdot z_{ij} \right)_{i=1}^{n} : z_{ij} \ge 0, \sum_{j=1}^{n} z_{ij} = 1 \ \forall i, \sum_{i=1}^{n} z_{ij} = 1 \ \forall j \right\}$$

[GOEMANS 2009] SIZE $O(n \log n)$

EXAMPLE 1 : PERMUTAHEDRON

$$\mathrm{PERM}(n) := \mathrm{conv}\left\{ (\sigma(1), \dots, \sigma(n)) : \sigma : [n] \to [n] \text{ BIJECTION} \right\}$$

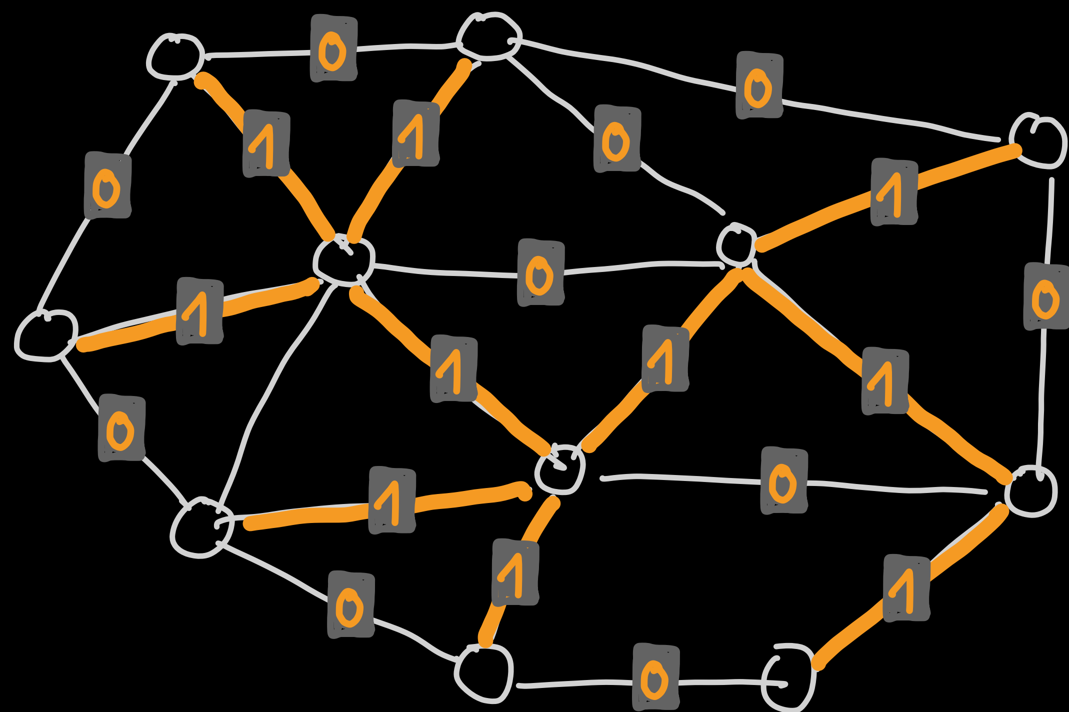
[WOLSEY 86]
$$= \left\{ \left(\sum_{j=1}^{n} y_{ij} \right)_{i=1}^{n} : 0 \le y_{ij} \underset{(i \neq j)}{=} 1 - y_{ji} \le 1 \right\}$$

[BIRKHOFF]
[V. NEUMANN]
$$= \left\{ \left(\sum_{j=1}^{n} j \cdot z_{ij} \right)_{i=1}^{n} : z_{ij} \ge 0 , \sum_{j=1}^{n} z_{ij} = 1 \,\forall i , \sum_{i=1}^{n} z_{ij} = 1 \,\forall j \right\}$$

[GOEMANS 2009] SIZE $O(n \log n)$

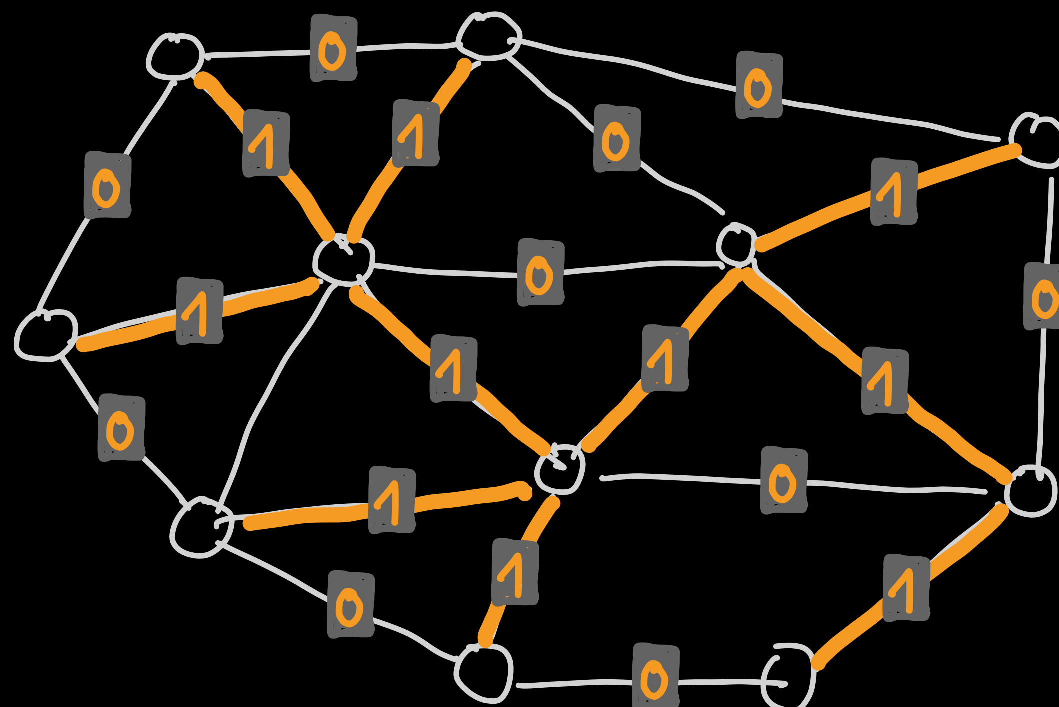
[K & PASHKOVICH 2012] FINITE REFLECTION GROUPS

EXAMPLE 2: SPANNING TREE POLYTOPES



$$\mathrm{CONV}\{\chi(T) : T \text{ SPANNING TREE IN } G\}$$

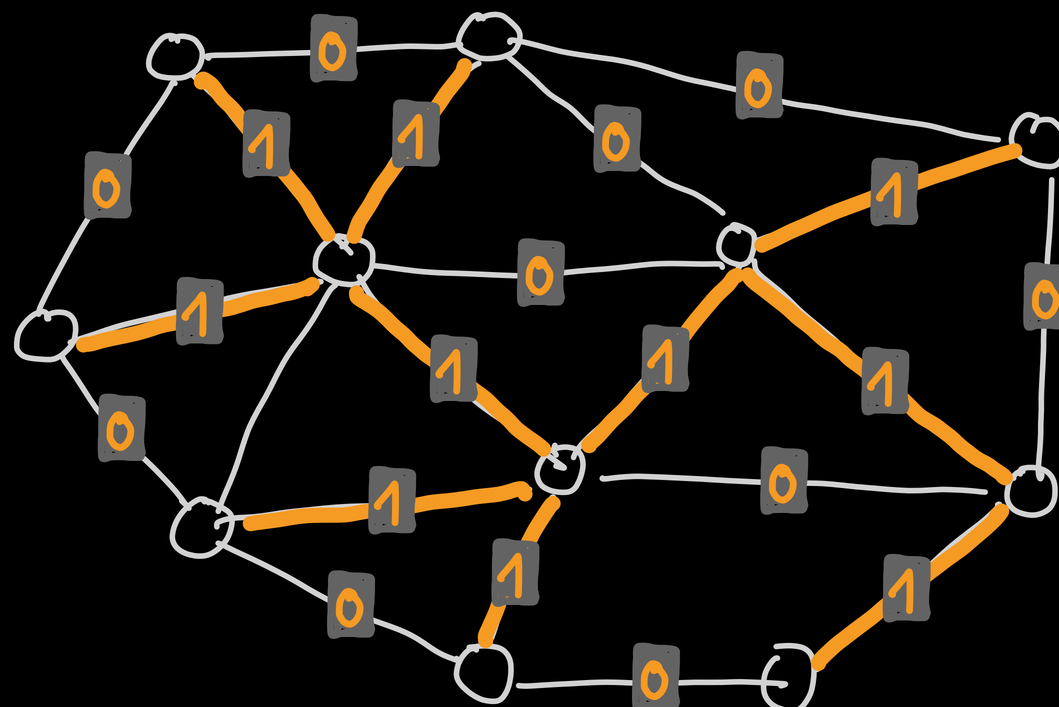
EXAMPLE 2: SPANNING TREE POLYTOPES



CONV $\{\chi(T) : T$ SPANNING TREE IN $G\}$

[MARTIN 90] $\qquad O(|V| \cdot |E|)$

EXAMPLE 2: SPANNING TREE POLYTOPES



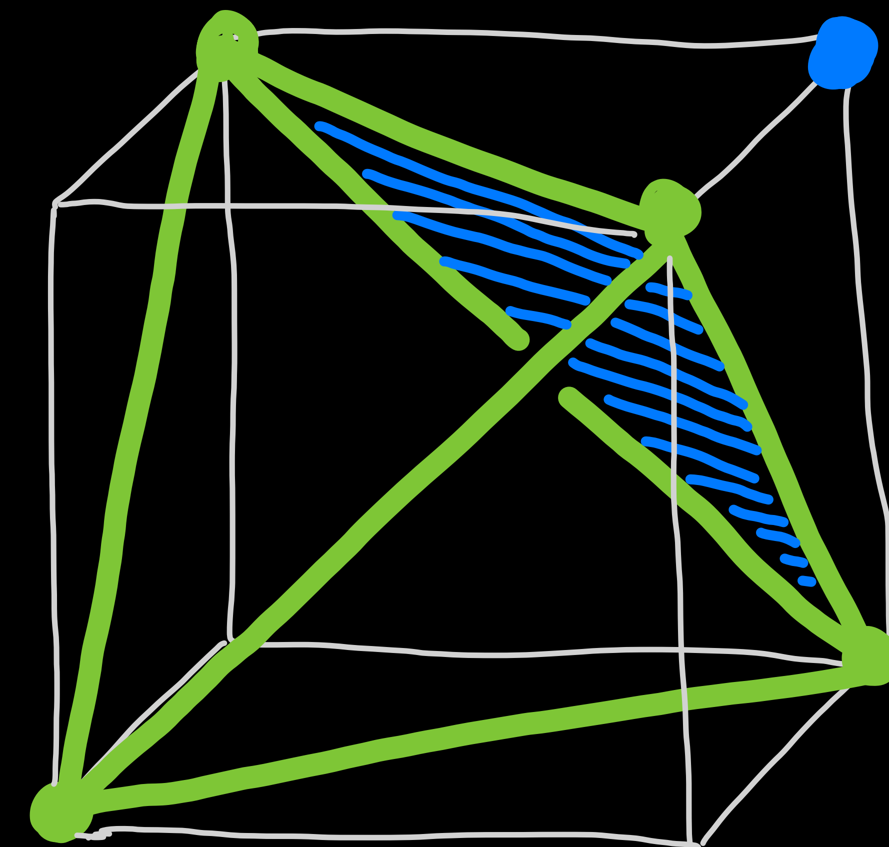
CONV $\{\chi(T) : T$ SPANNING TREE IN $G\}$

[MARTIN 90] $O(|V| \cdot |E|)$

[WILLIAMS 01] $O(|V|)$ FOR PLANAR G

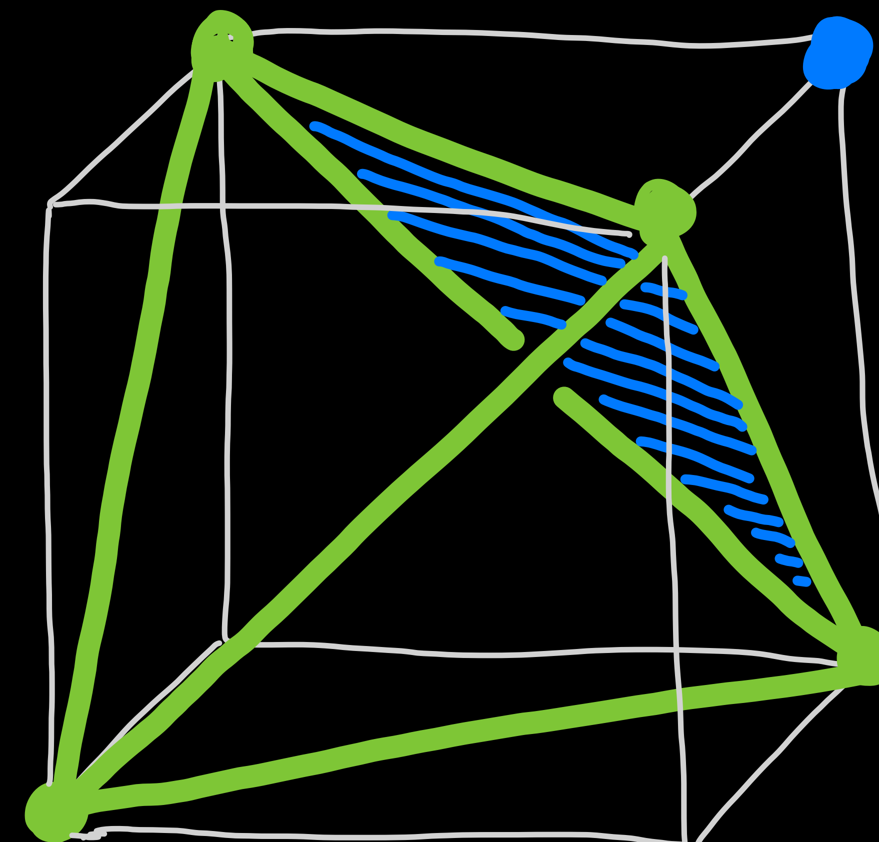
EXAMPLE 3: PARITY-POLYTOPE

$$\operatorname{conv}\{x \in \{0,1\}^n : \mathbb{1}^T x \in 2\mathbb{Z}\}$$



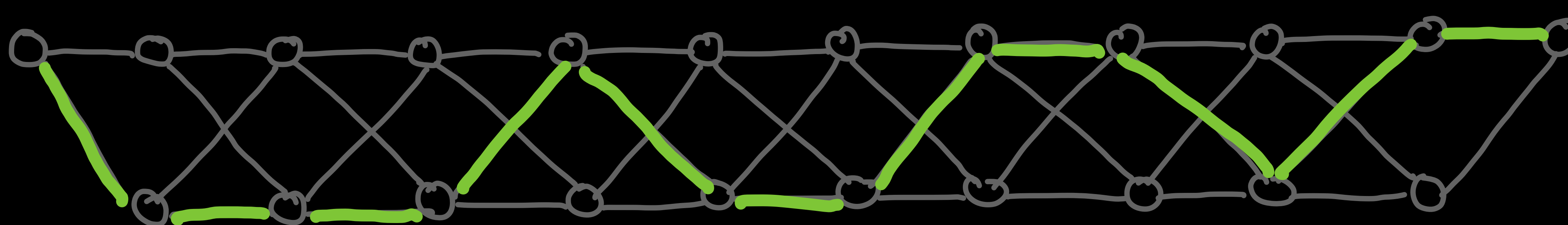
EXAMPLE 3: PARITY-POLYTOPE

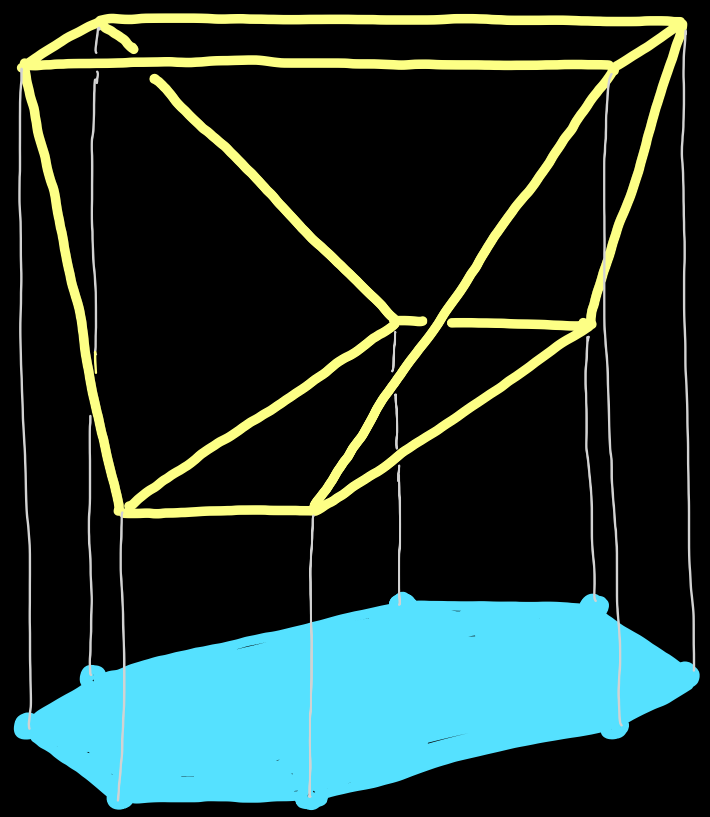
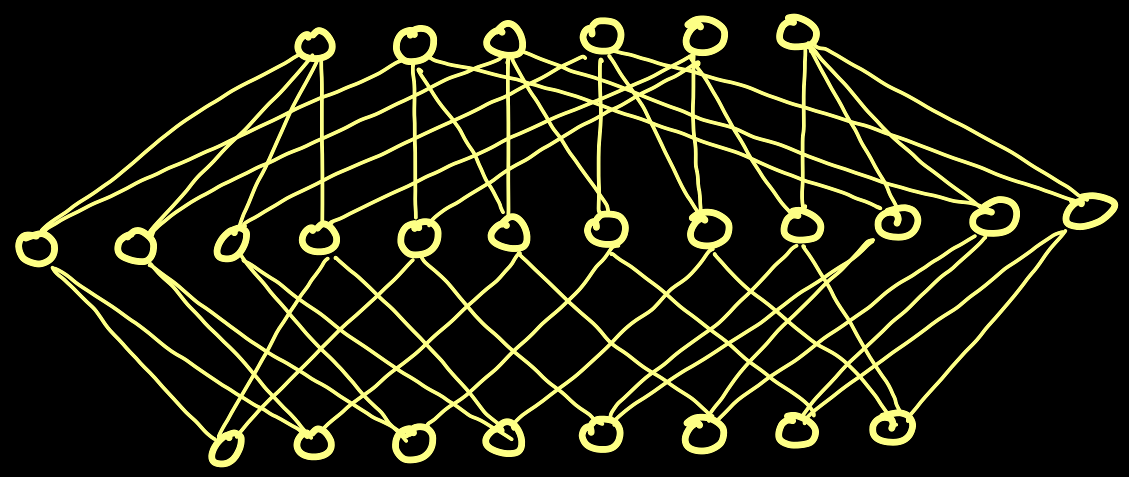
$$\mathrm{conv}\{x \in \{0,1\}^n : \mathbb{1}^T x \in 2\mathbb{Z}\}$$

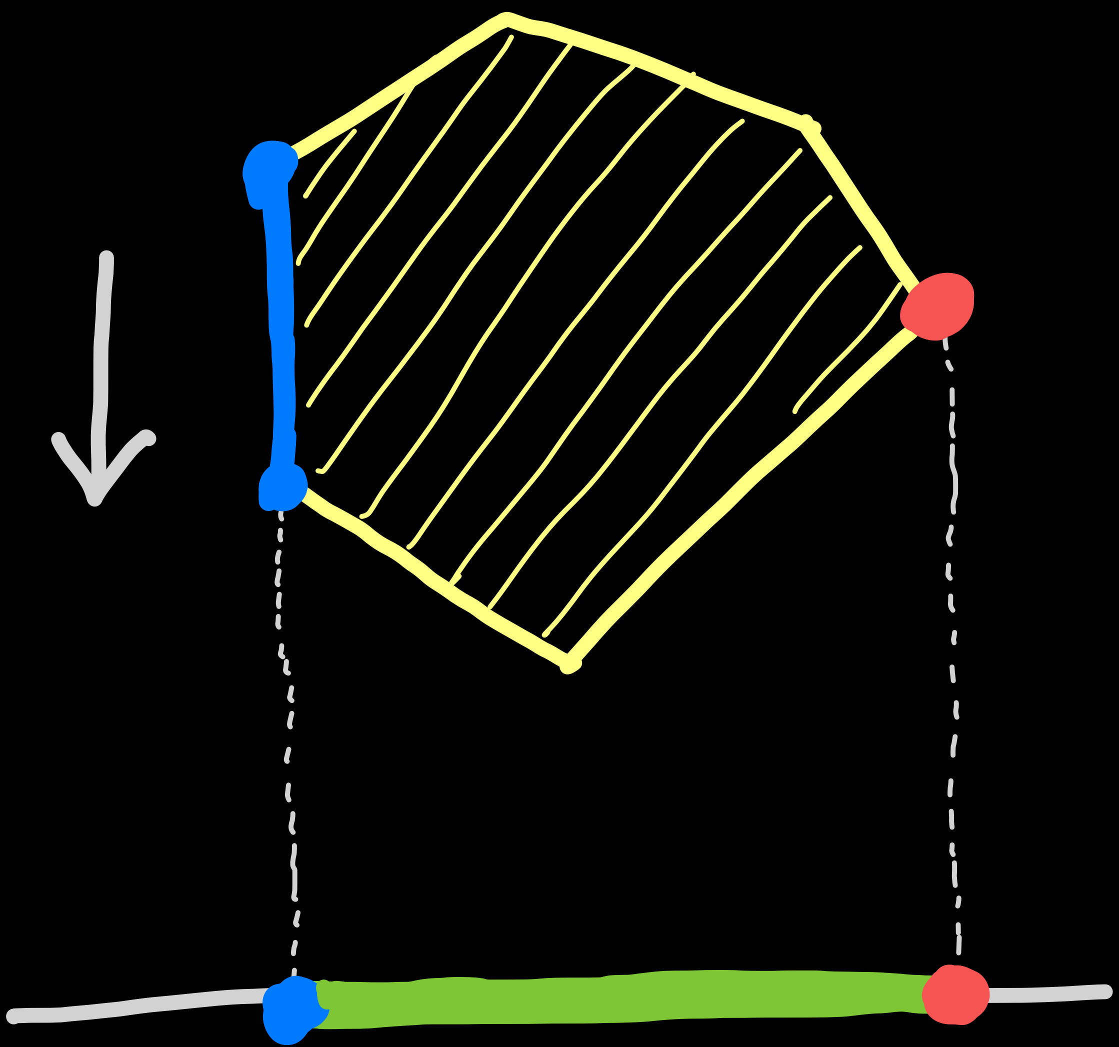


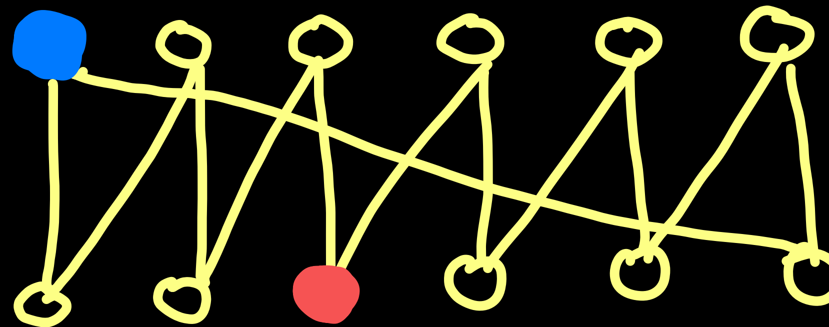
[CARR & KONJEVOD 04]:

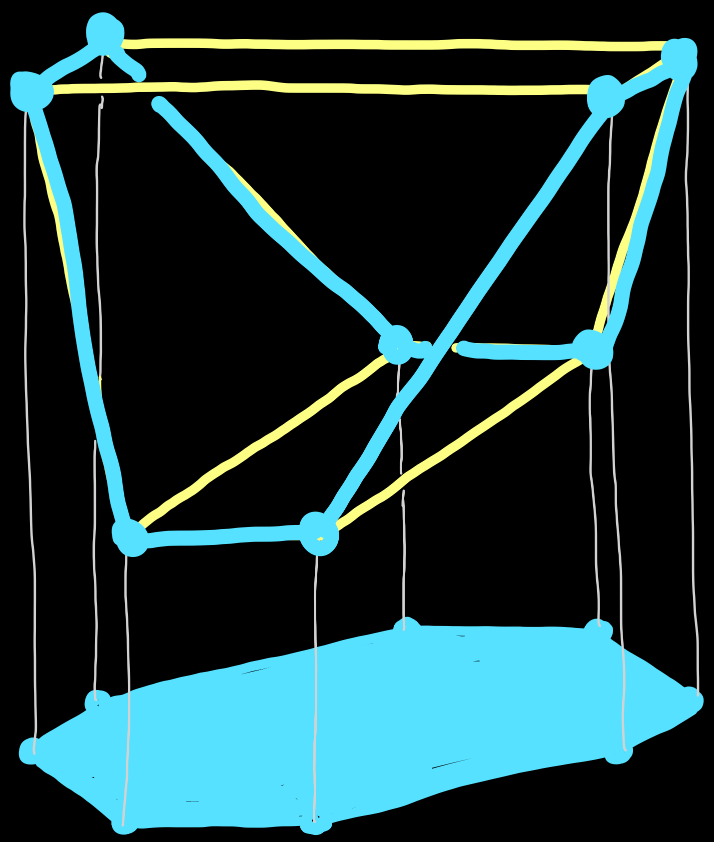
$\leq 4 \cdot n$

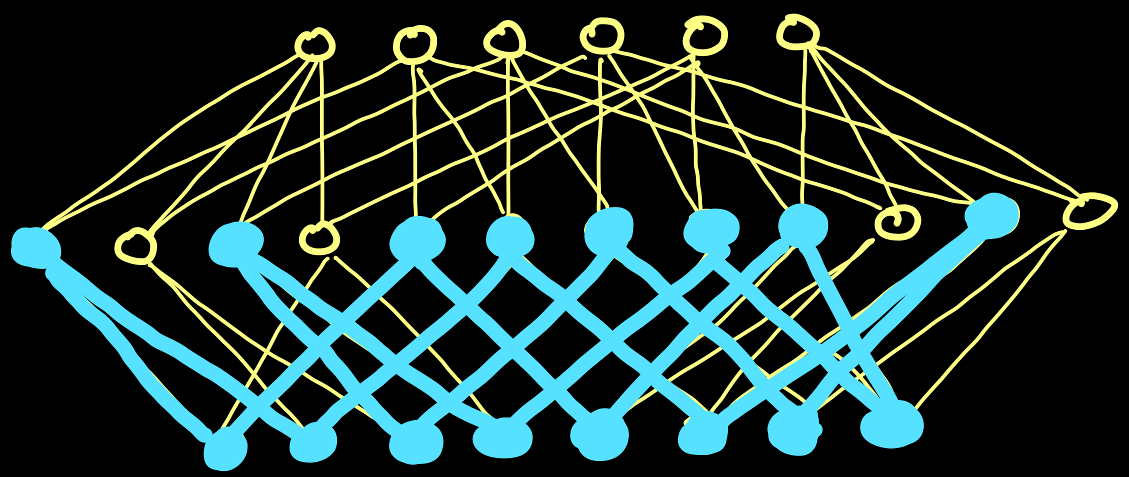


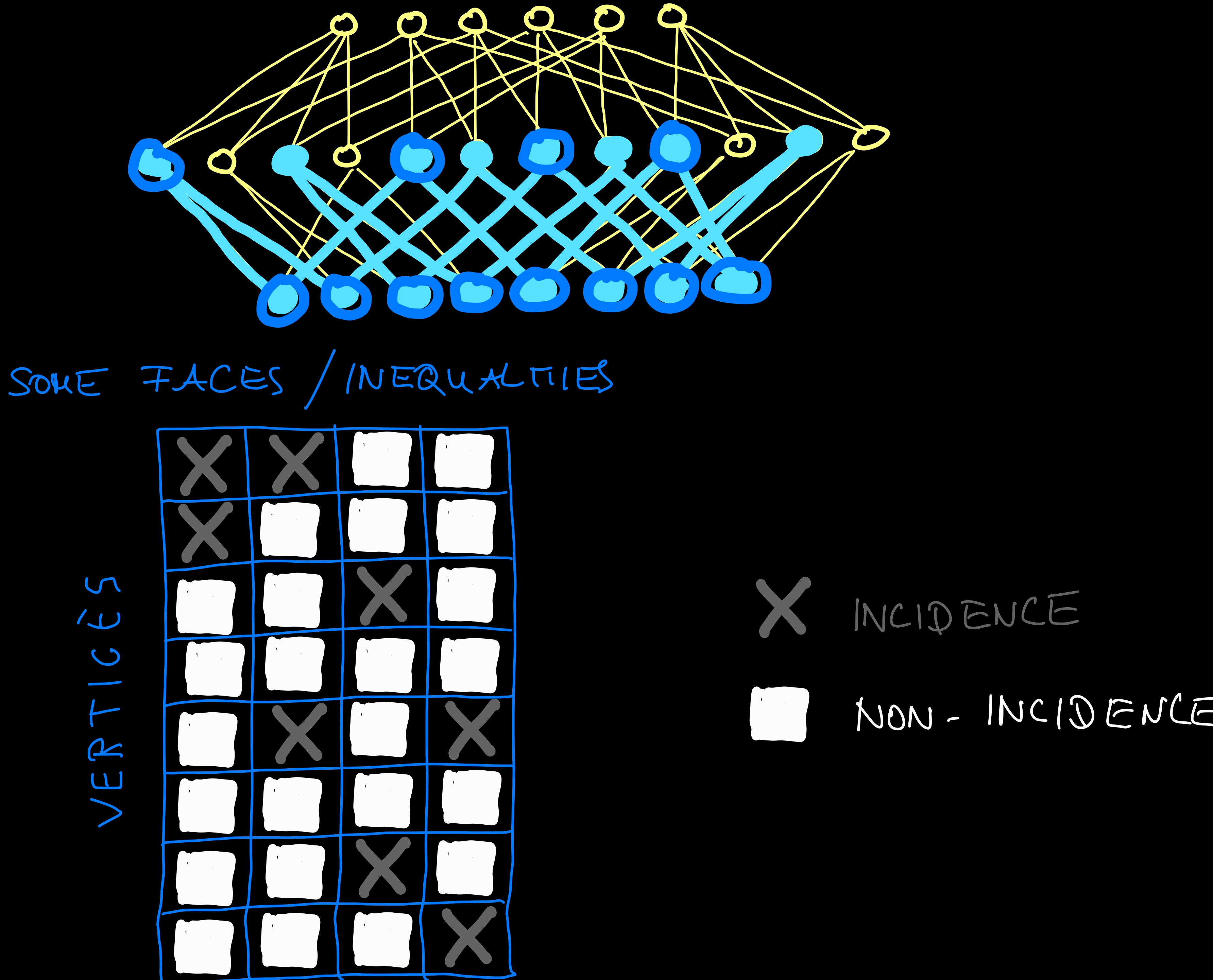



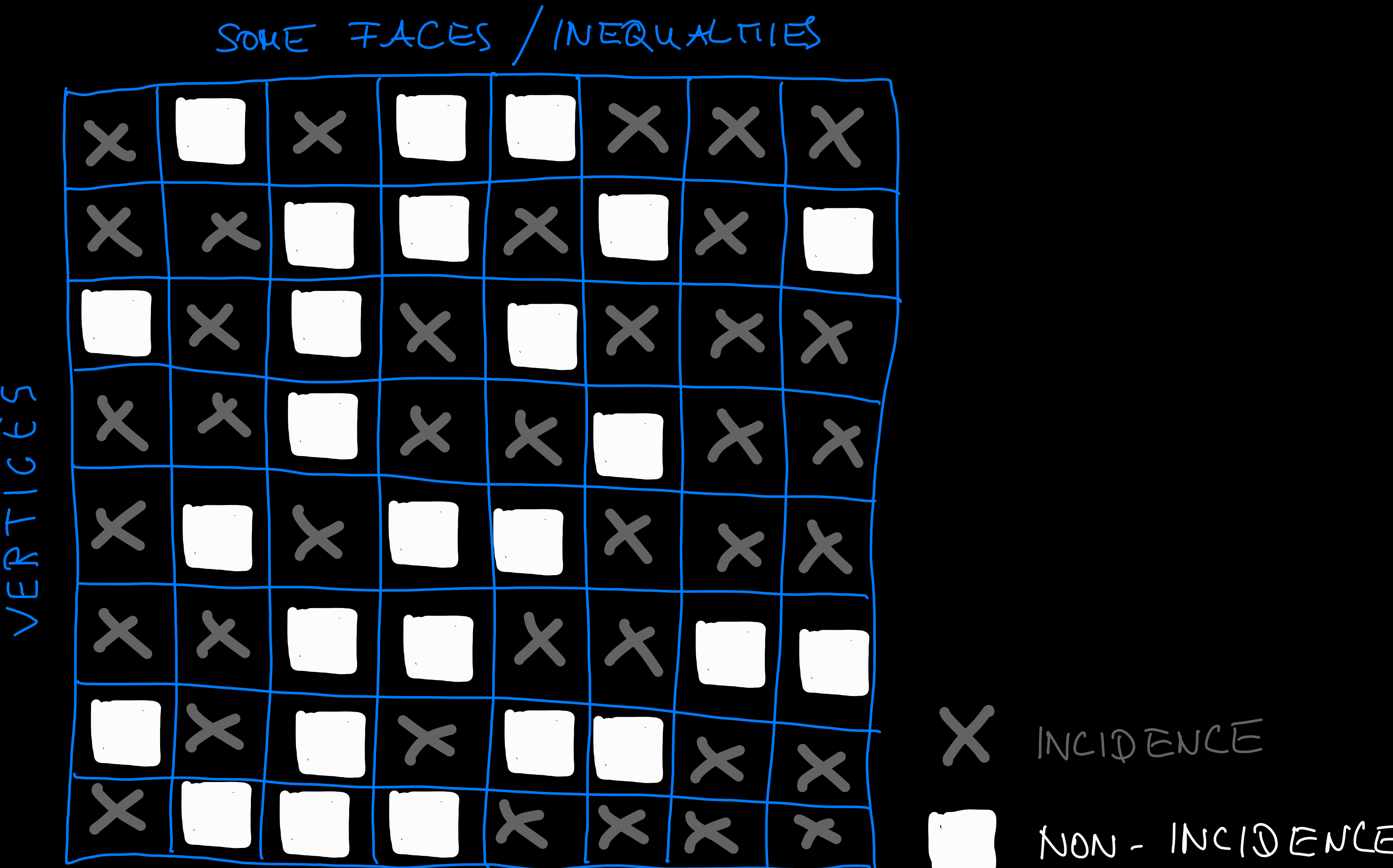


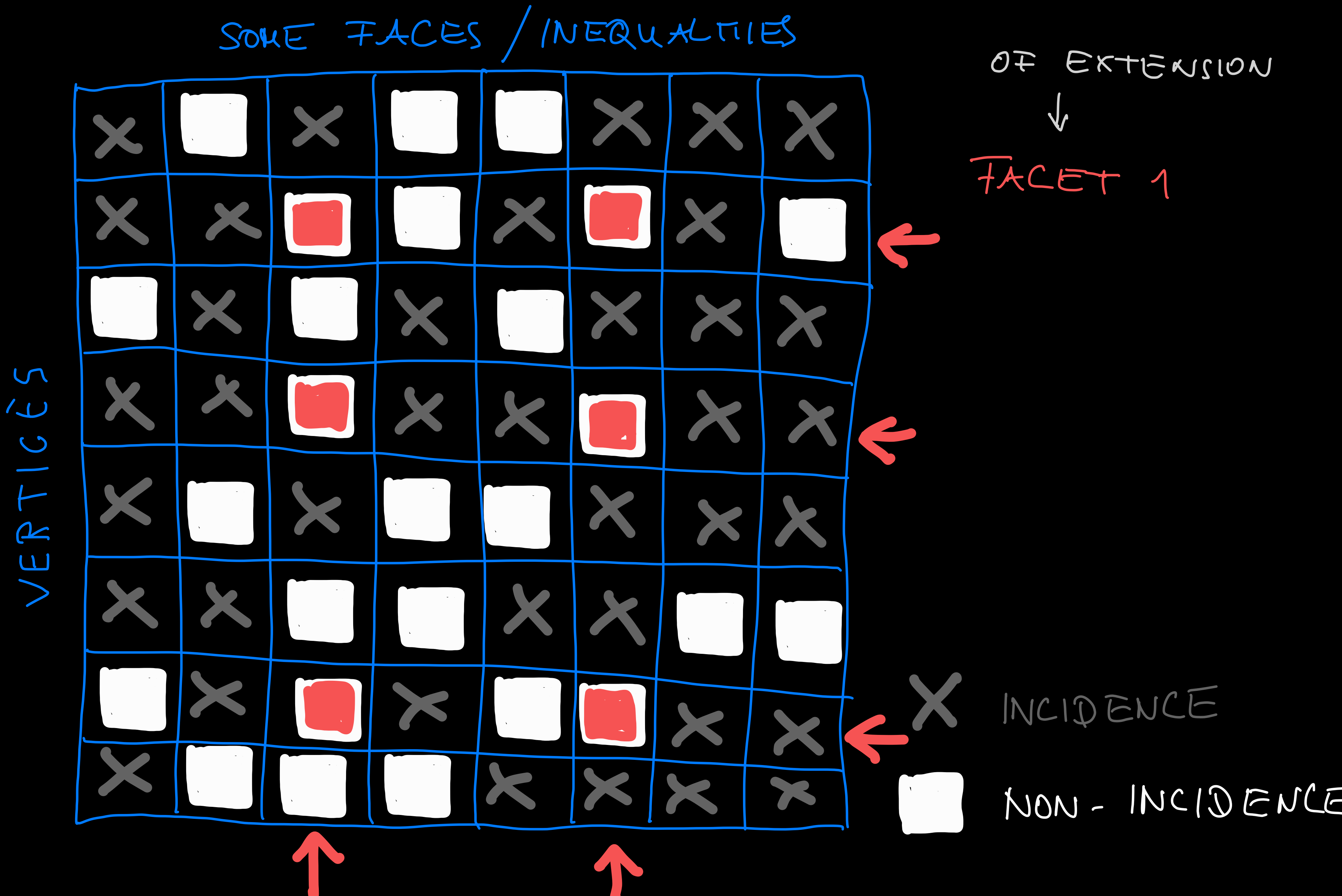


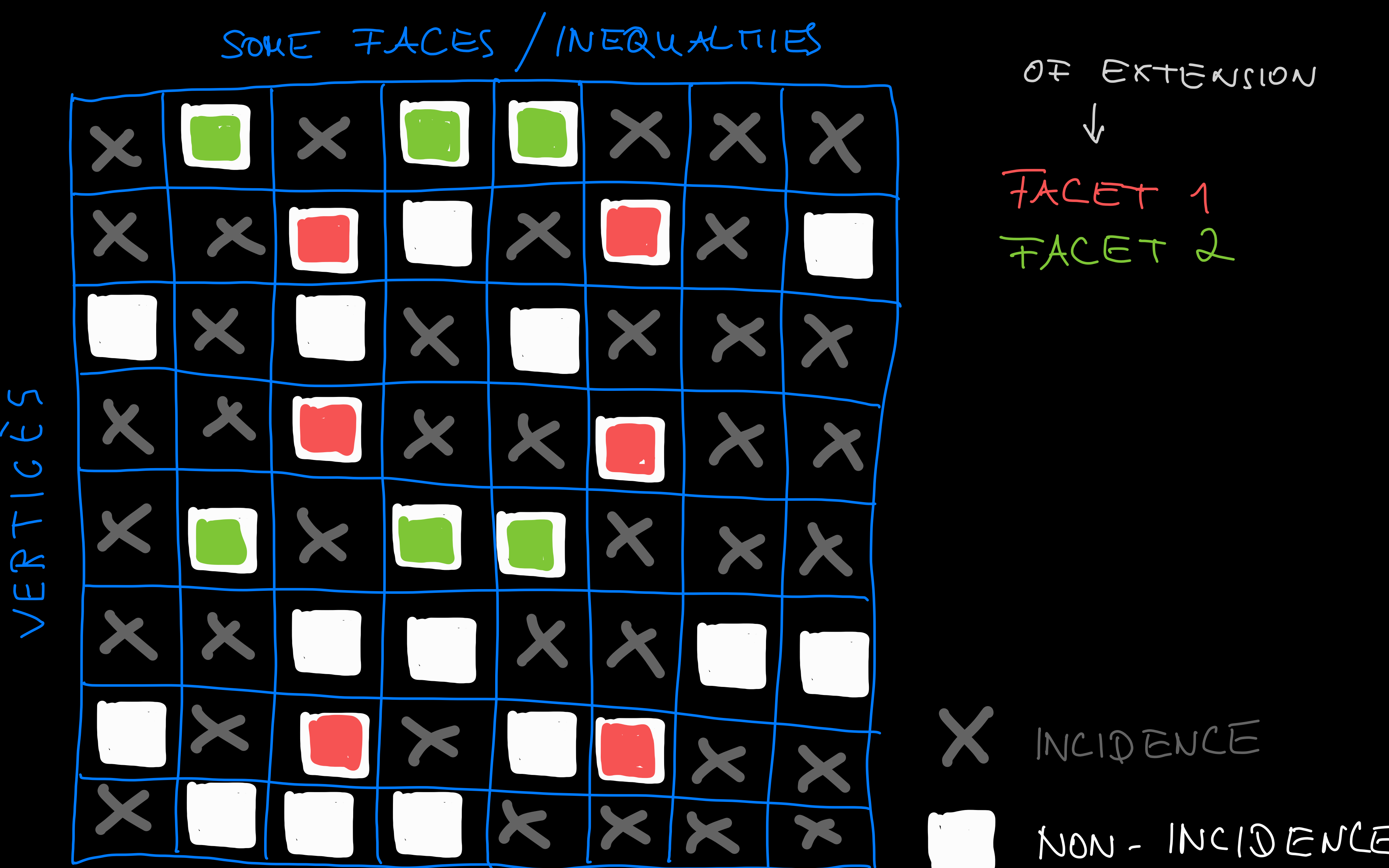


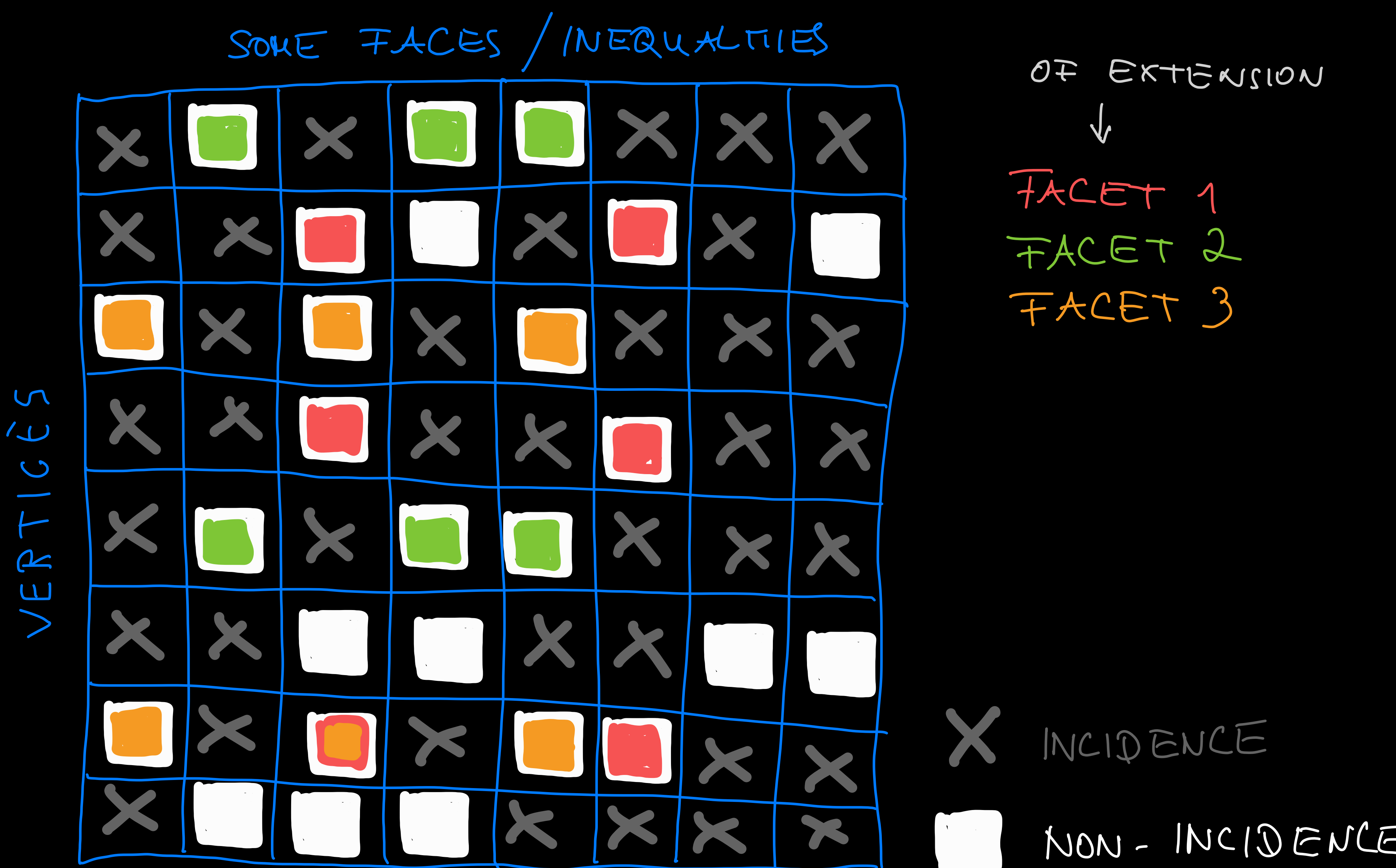


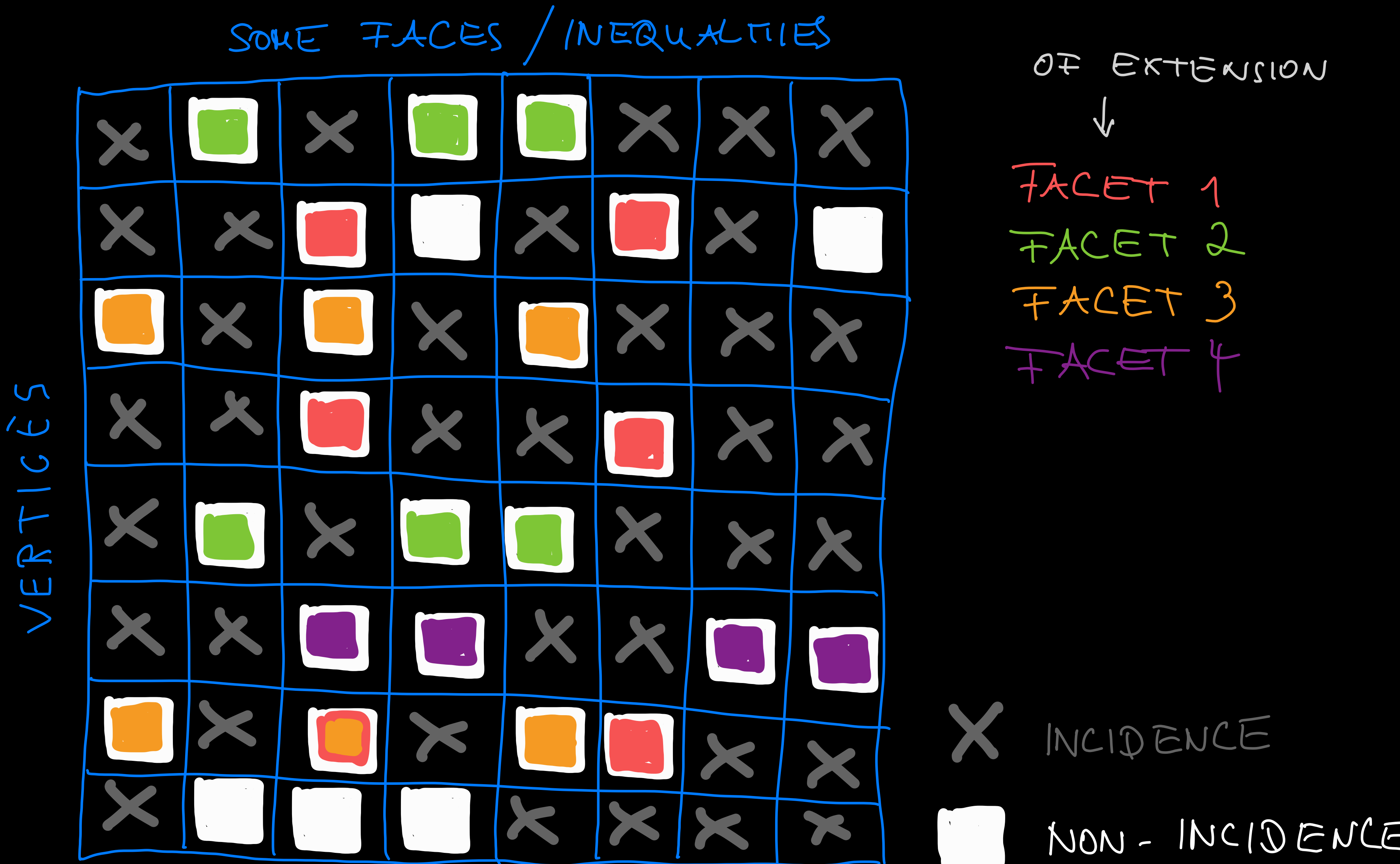

SOME FACES / INEQUALITIES
VERTICES
INCIDENCE
NON - INCIDENCE

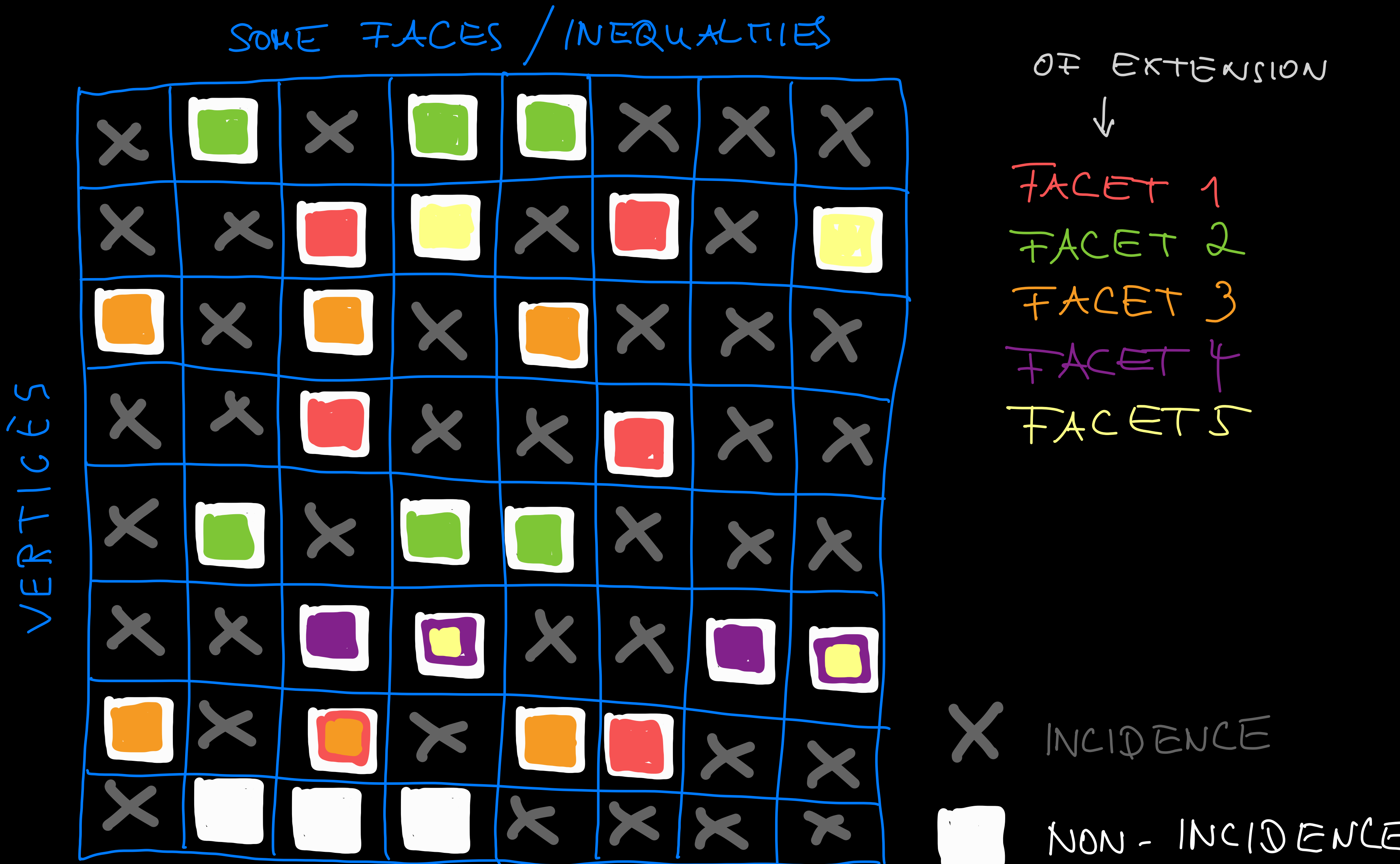

SOME FACES / INEQUALITIES
VERTICES
INCIDENCE
NON-INCIDENCE

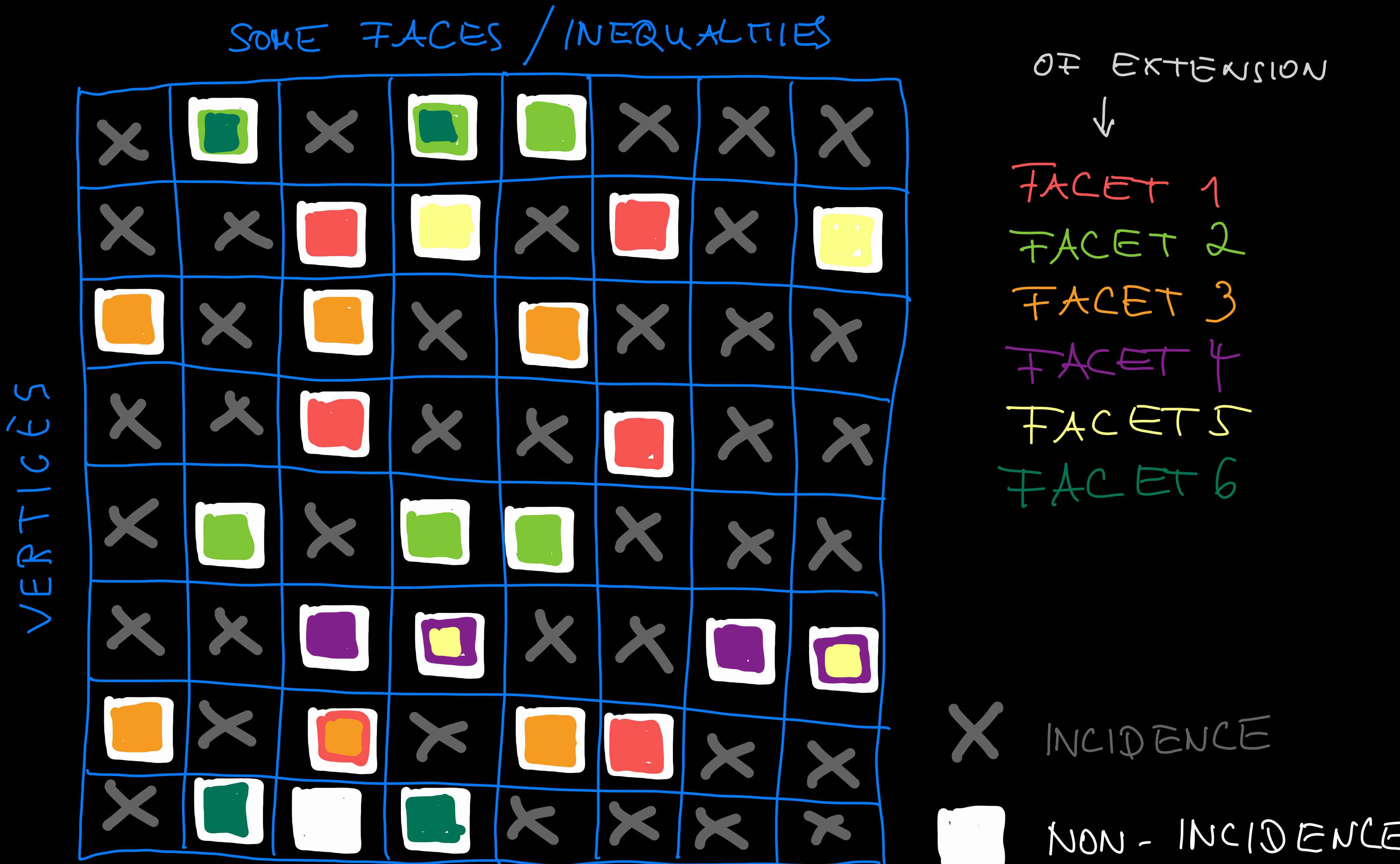

SOME FACES / INEQUALITIES
OF EXTENSION
FACET 1
VERTICES
INCIDENCE
NON-INCIDENCE

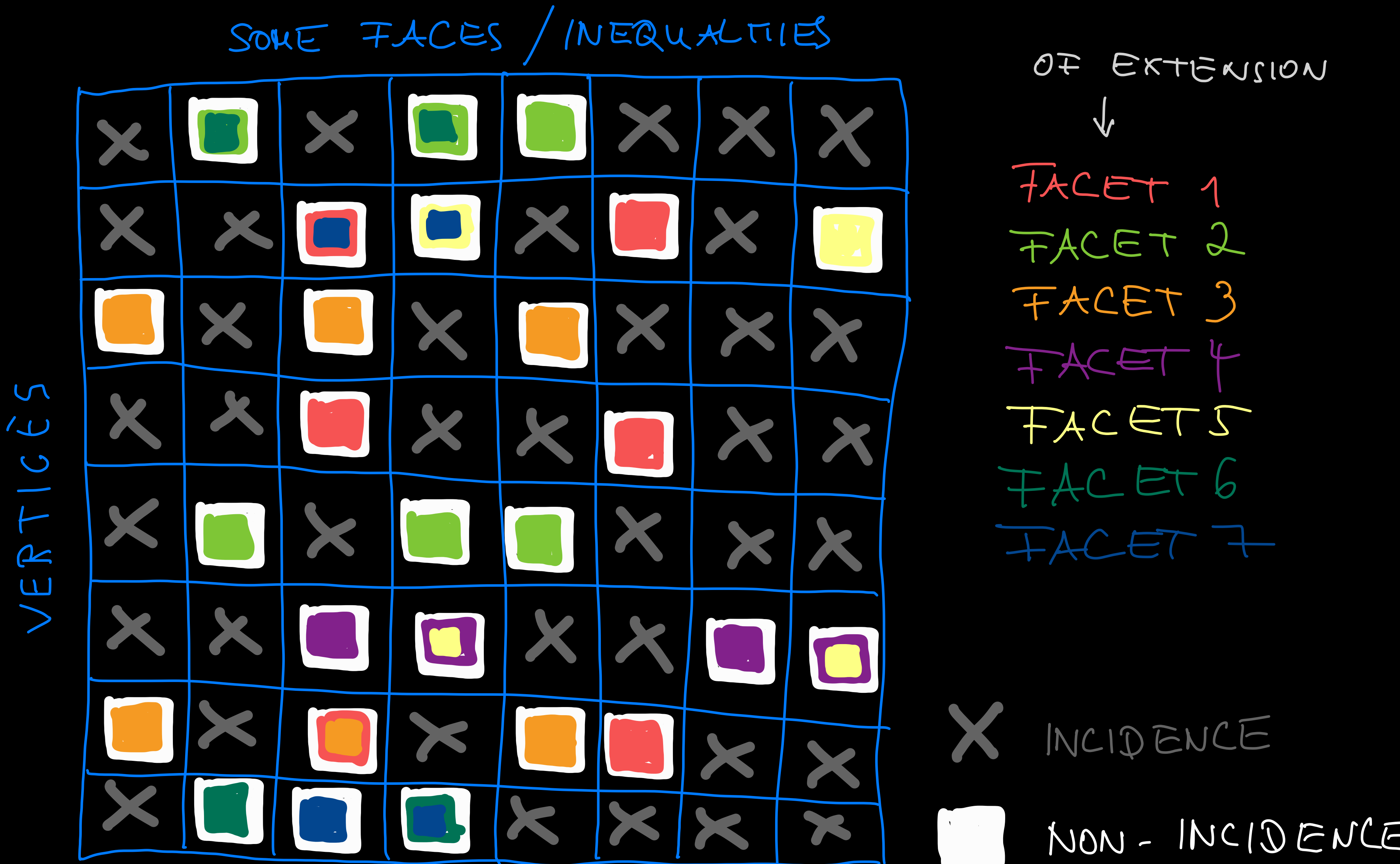

SOME FACES / INEQUALITIES
VERTICES
OF EXTENSION
FACET 1
FACET 2
INCIDENCE
NON-INCIDENCE

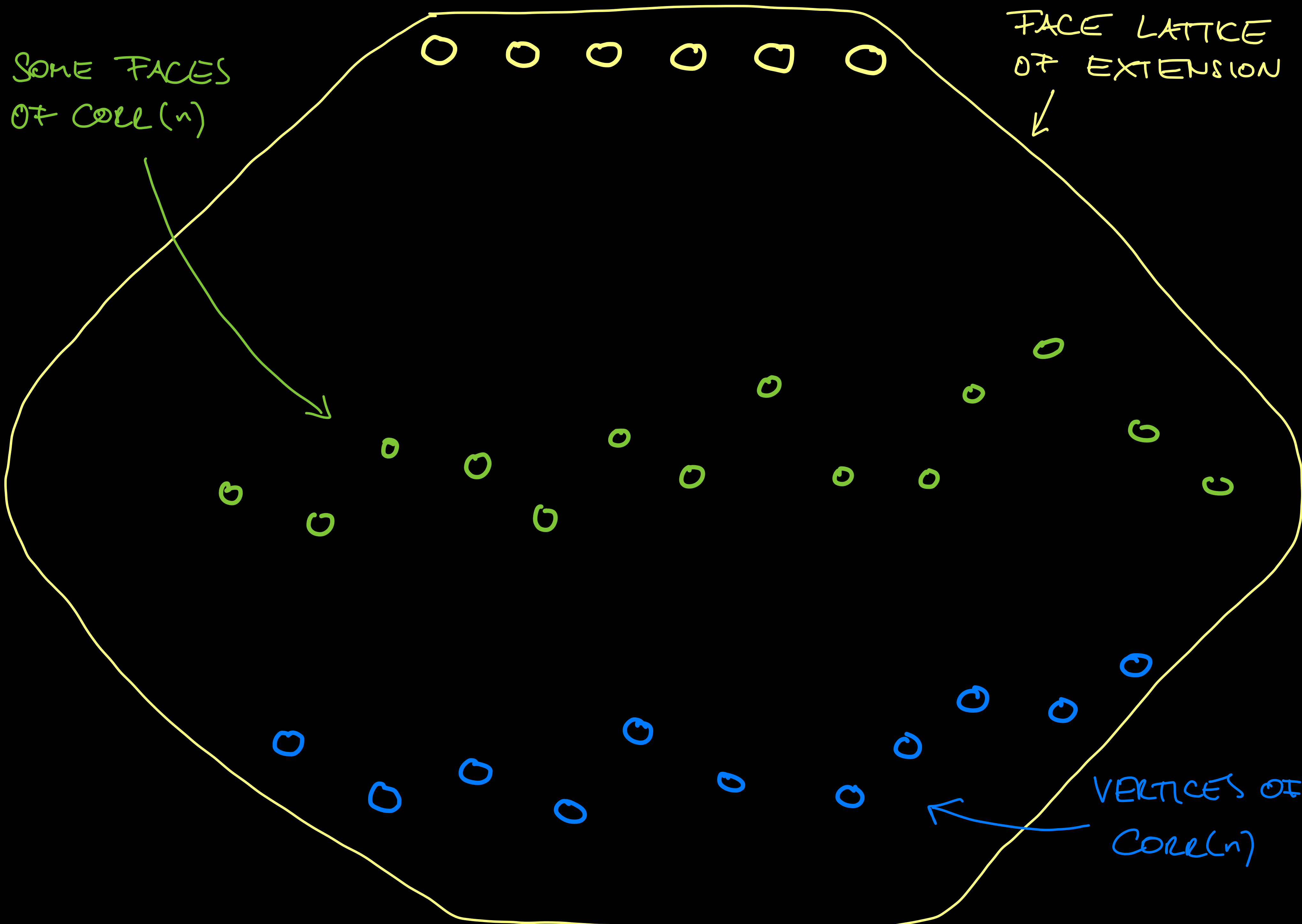

SOME FACES / INEQUALITIES
VERTICES
OF EXTENSION
FACET 1
FACET 2
FACET 3
INCIDENCE
NON-INCIDENCE

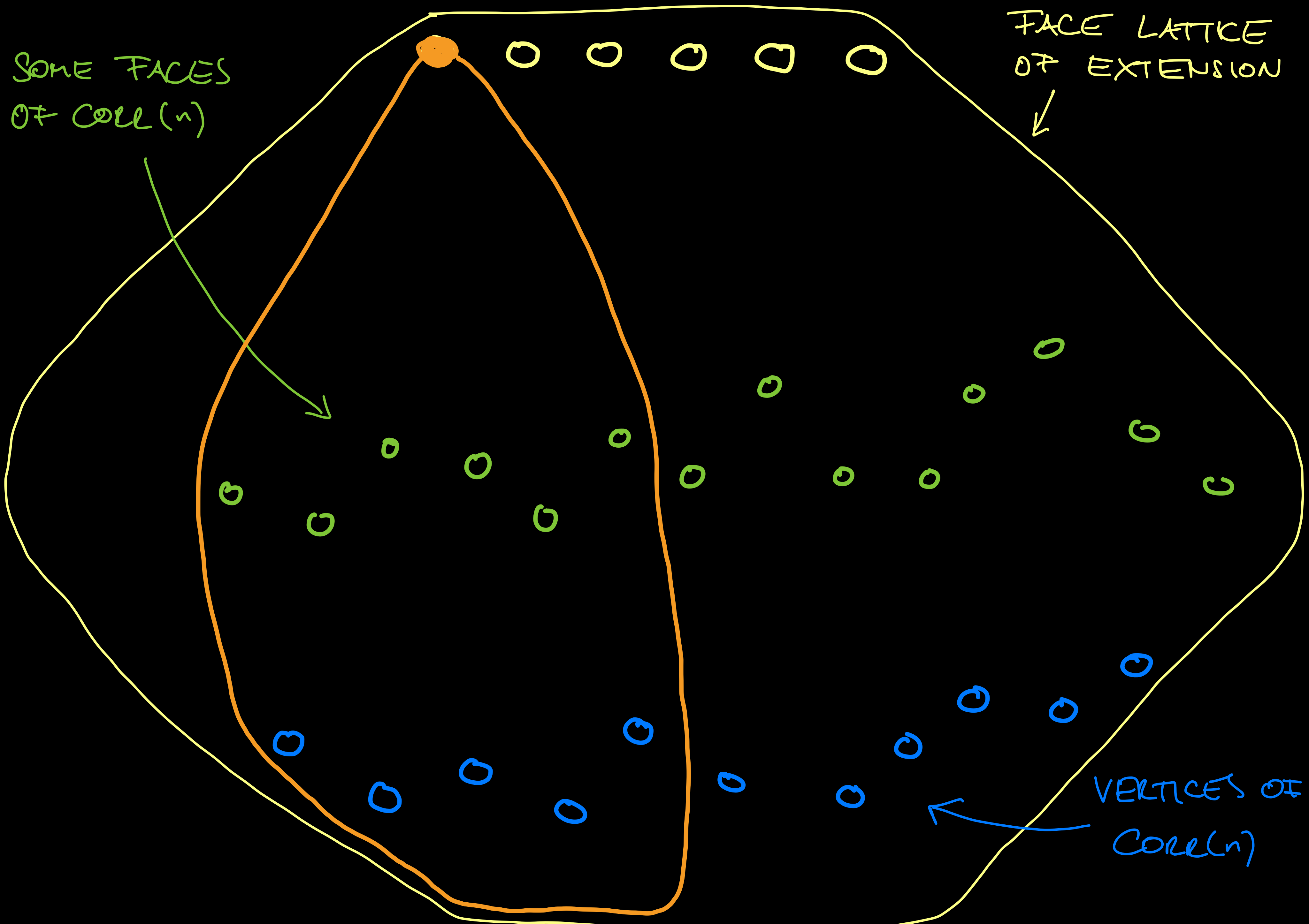

SOME FACES / INEQUALITIES
VERTICES
OF EXTENSION
FACET 1
FACET 2
FACET 3
FACET 4
INCIDENCE
NON-INCIDENCE


SOME FACES / INEQUALITIES
VERTICES
OF EXTENSION
FACET 1
FACET 2
FACET 3
FACET 4
FACET 5
INCIDENCE
NON-INCIDENCE

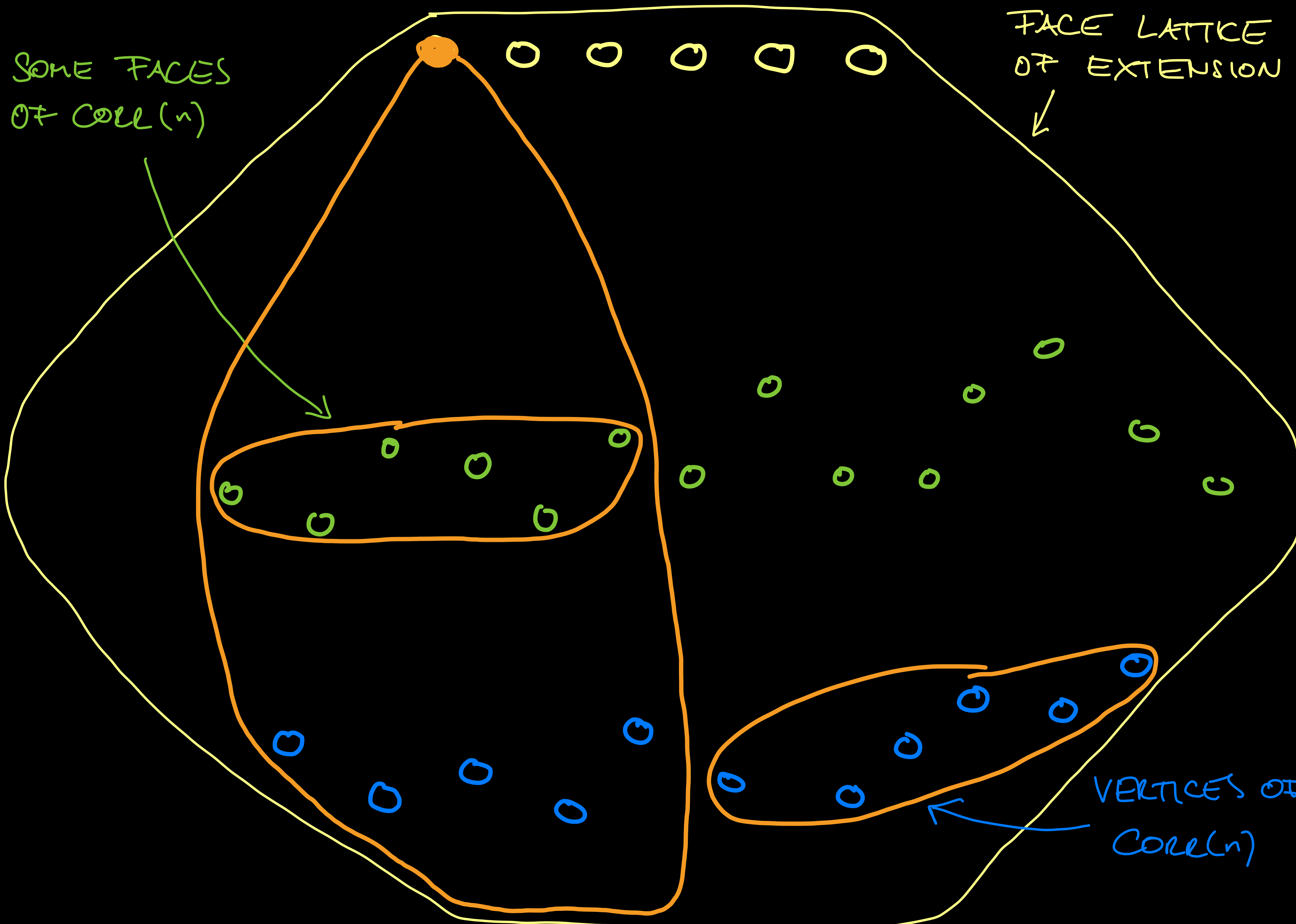

SOME FACES / INEQUALITIES
VERTICES
OF EXTENSION
FACET 1
FACET 2
FACET 3
FACET 4
FACET 5
FACET 6
INCIDENCE
NON-INCIDENCE

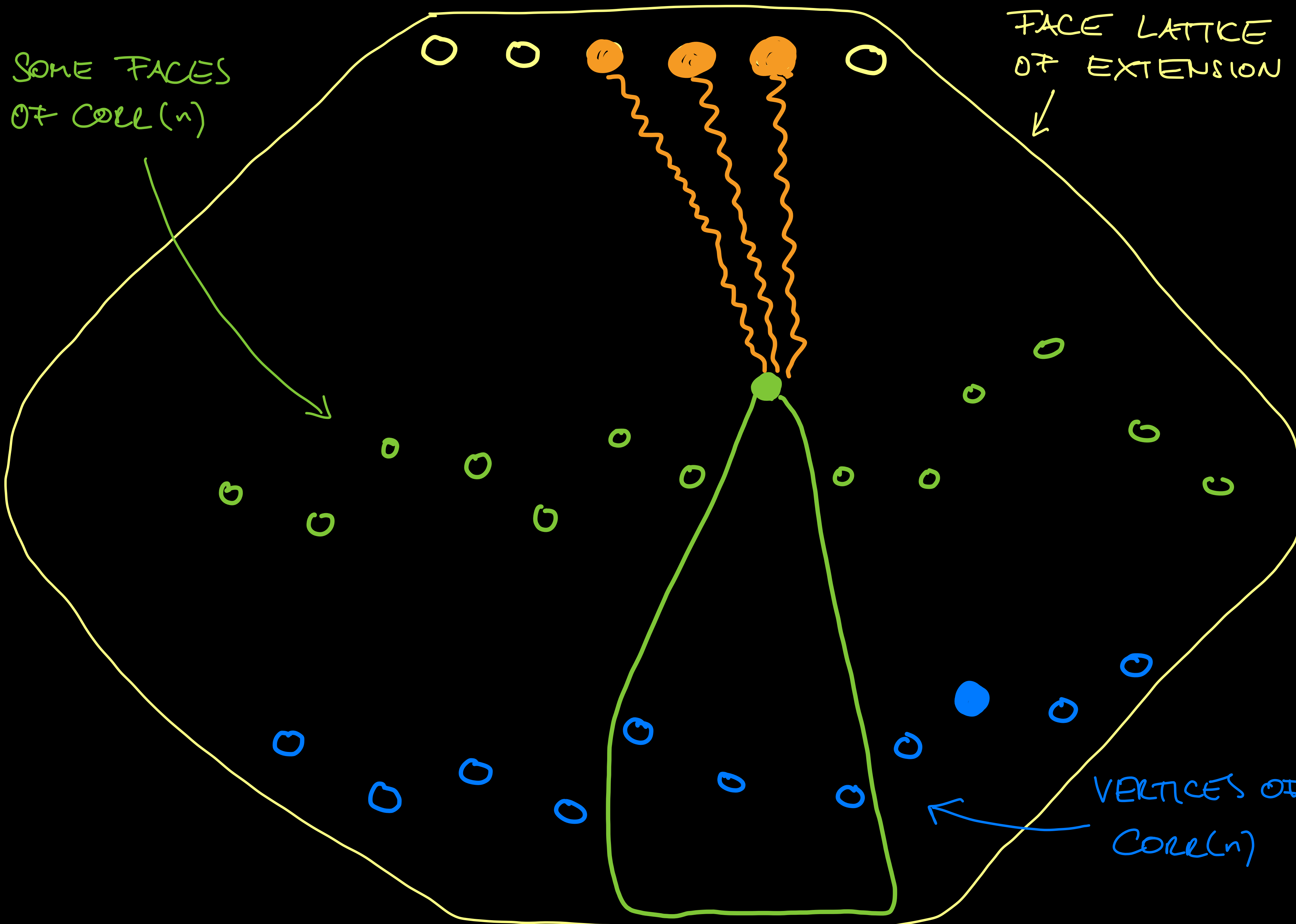

SOME FACES / INEQUALITIES
VERTICES
OF EXTENSION
FACET 1
FACET 2
FACET 3
FACET 4
FACET 5
FACET 6
FACET 7
INCIDENCE
NON-INCIDENCE


Some faces of Corr(n)
Face lattice of extension
Vertices of Corr(n)



FACE LATTICE OF EXTENSION
SOME FACES OF CORR(n)
VERTICES OF CORR(n)

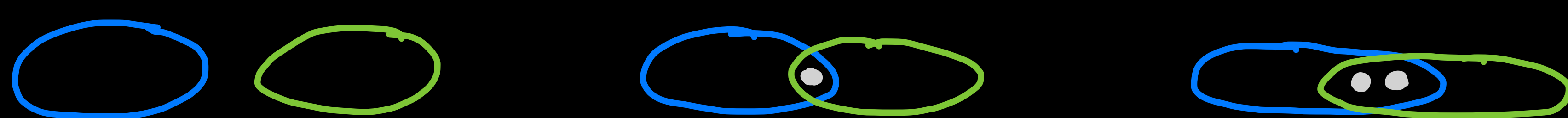

SOME FACES OF CORR(n)
FACE LATTICE OF EXTENSION
VERTICES OF CORR(n)


SOME FACES
OF CORR(n)
FACE LATTICE
OF EXTENSION
VERTICES OF
CORR(n)

$$\mathrm{CORR}(n) = \mathrm{conv}\left\{ \left(\chi(A), \chi(E(A)) \right) : A \subseteq [n] \right\}$$

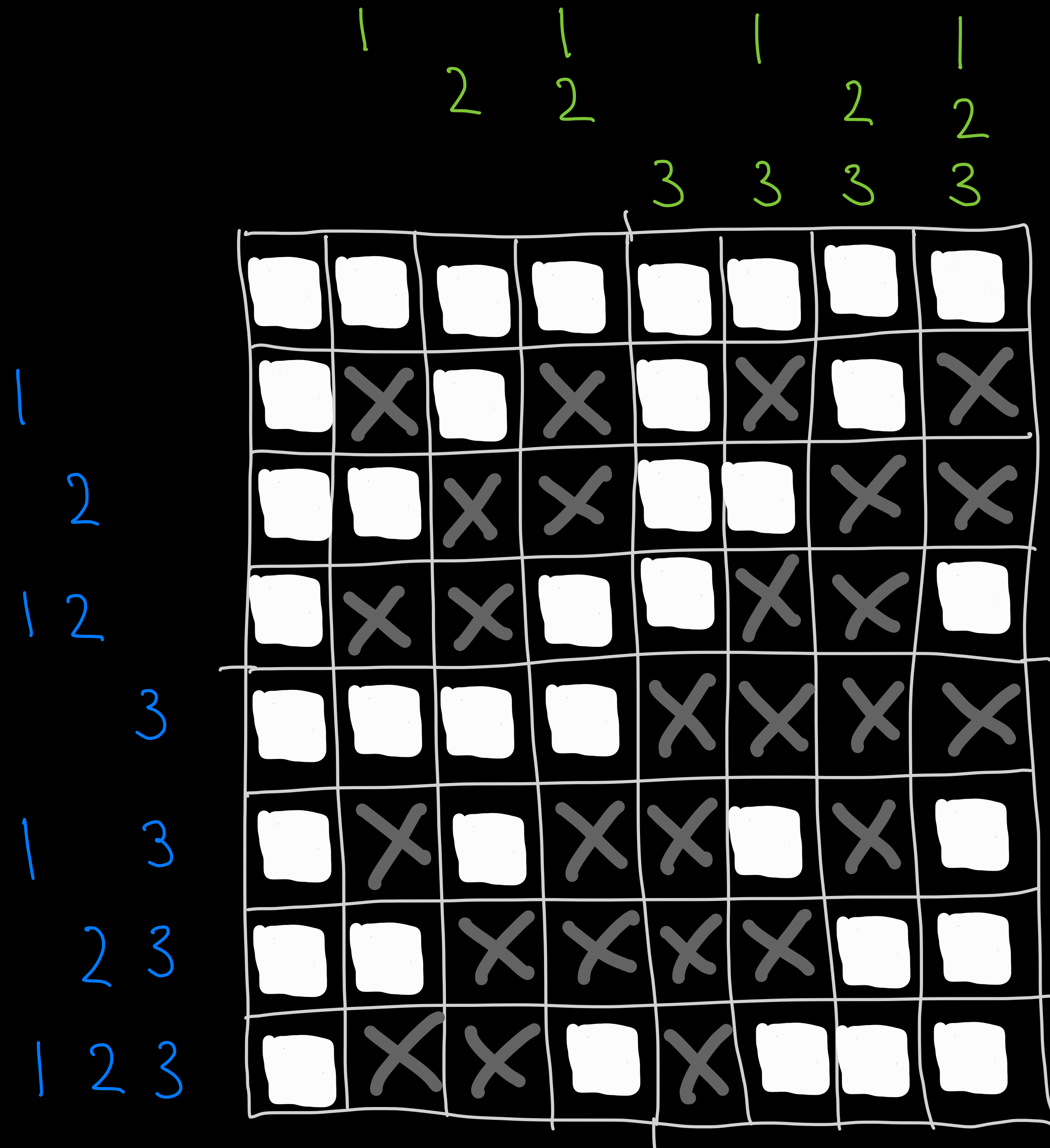
FOR EACH $B \subseteq [n]$:

$$\underbrace{\left\langle \left(\chi(B), -2 \cdot \chi(E(B)) \right), \left(\chi(A), \chi(E(A)) \right) \right\rangle}_{|A \cap B| - 2 \cdot |E(A \cap B)|} \leq 1$$



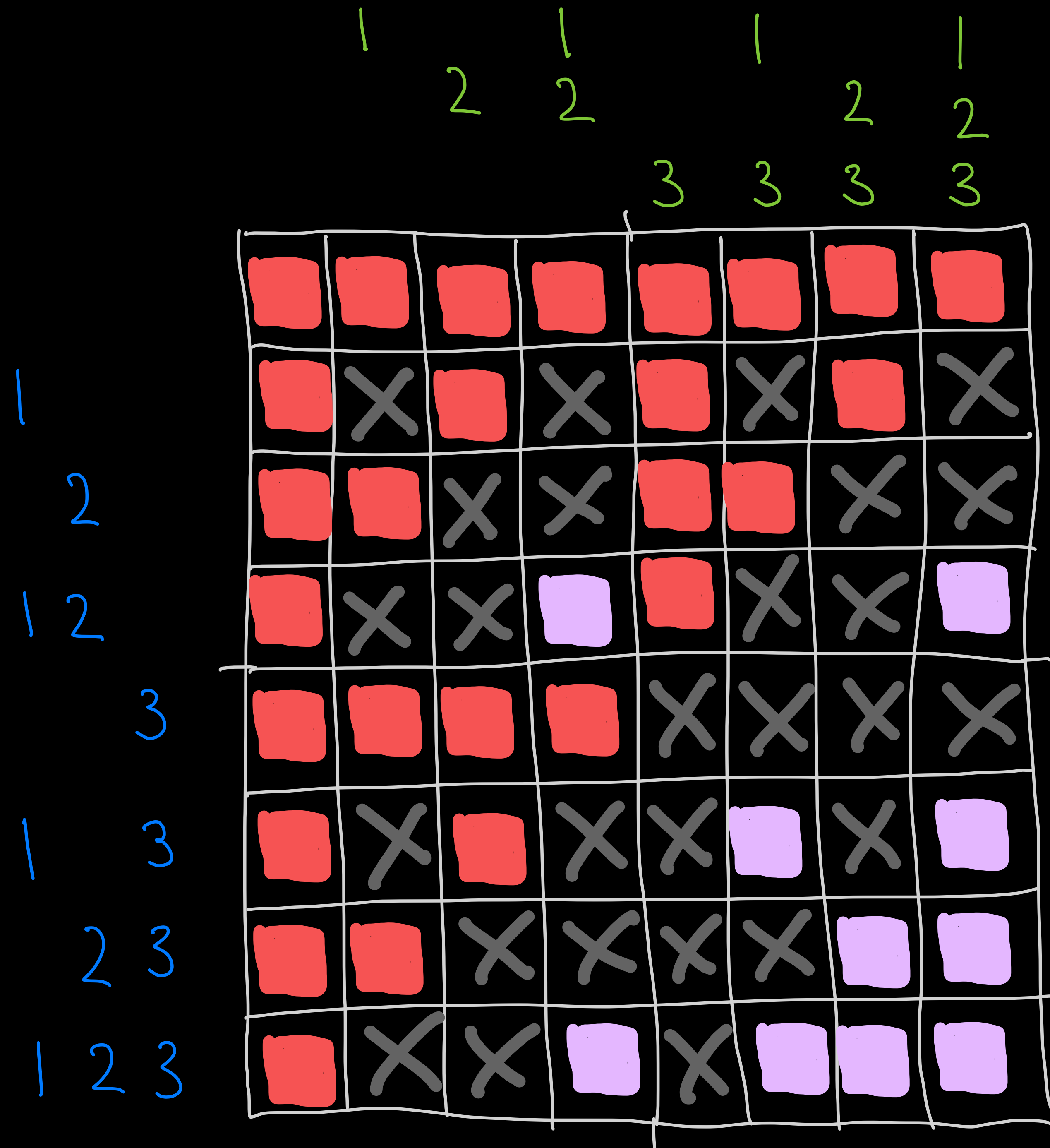
$|A \cap B| = 0$ | $|A \cap B| = 1$ | $|A \cap B| \geq 2$

NON-INCIDENT | INCIDENT | NON-INCIDENT



TO SHOW:

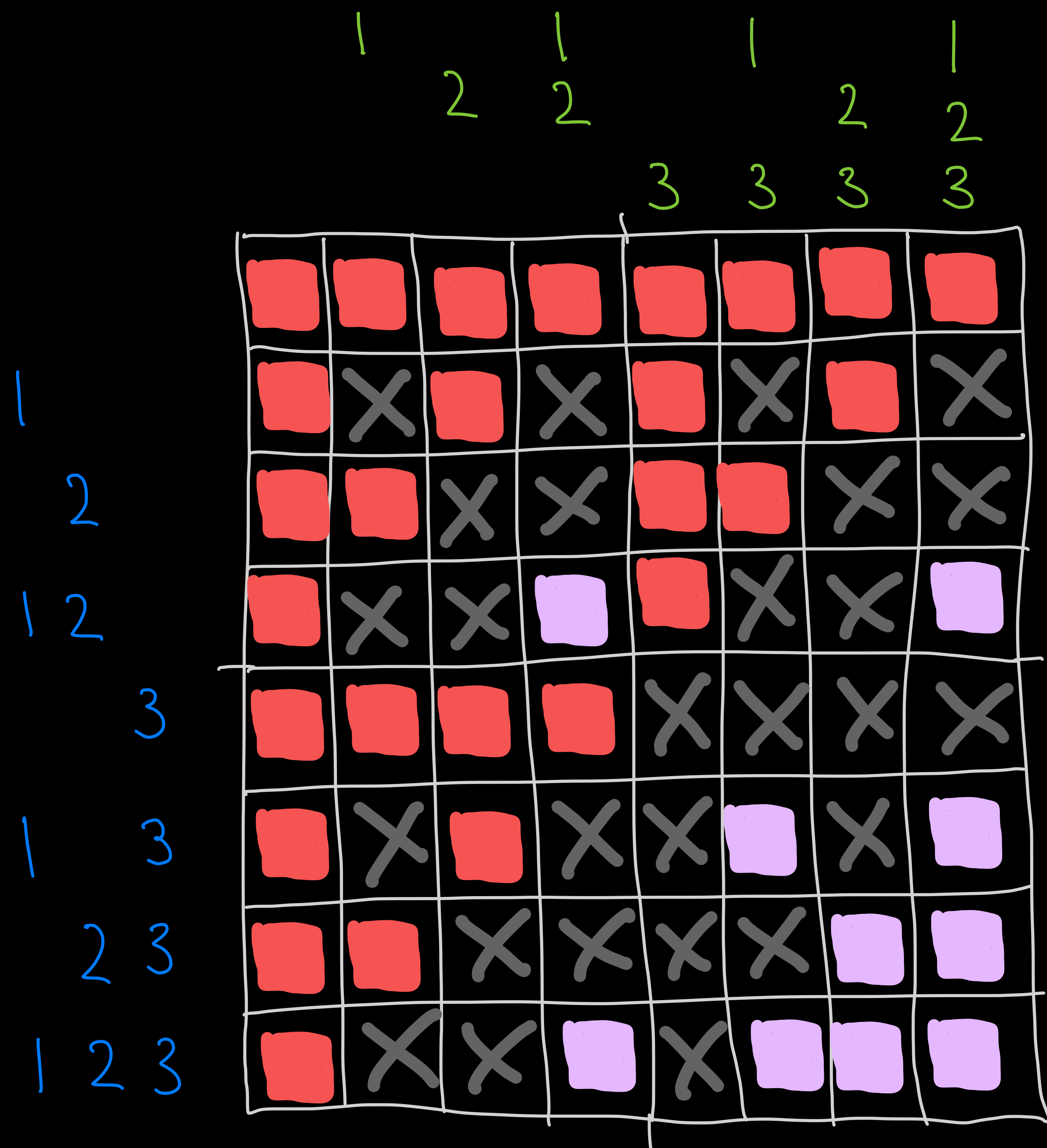
AT LEAST $\left(\frac{3}{2}\right)^n$ X-FREE RECTANGLES NECESSARY TO COVER ALL □



TO SHOW:

AT LEAST $\left(\frac{3}{2}\right)^n$ X-FREE RECTANGLES NECESSARY TO COVER ALL

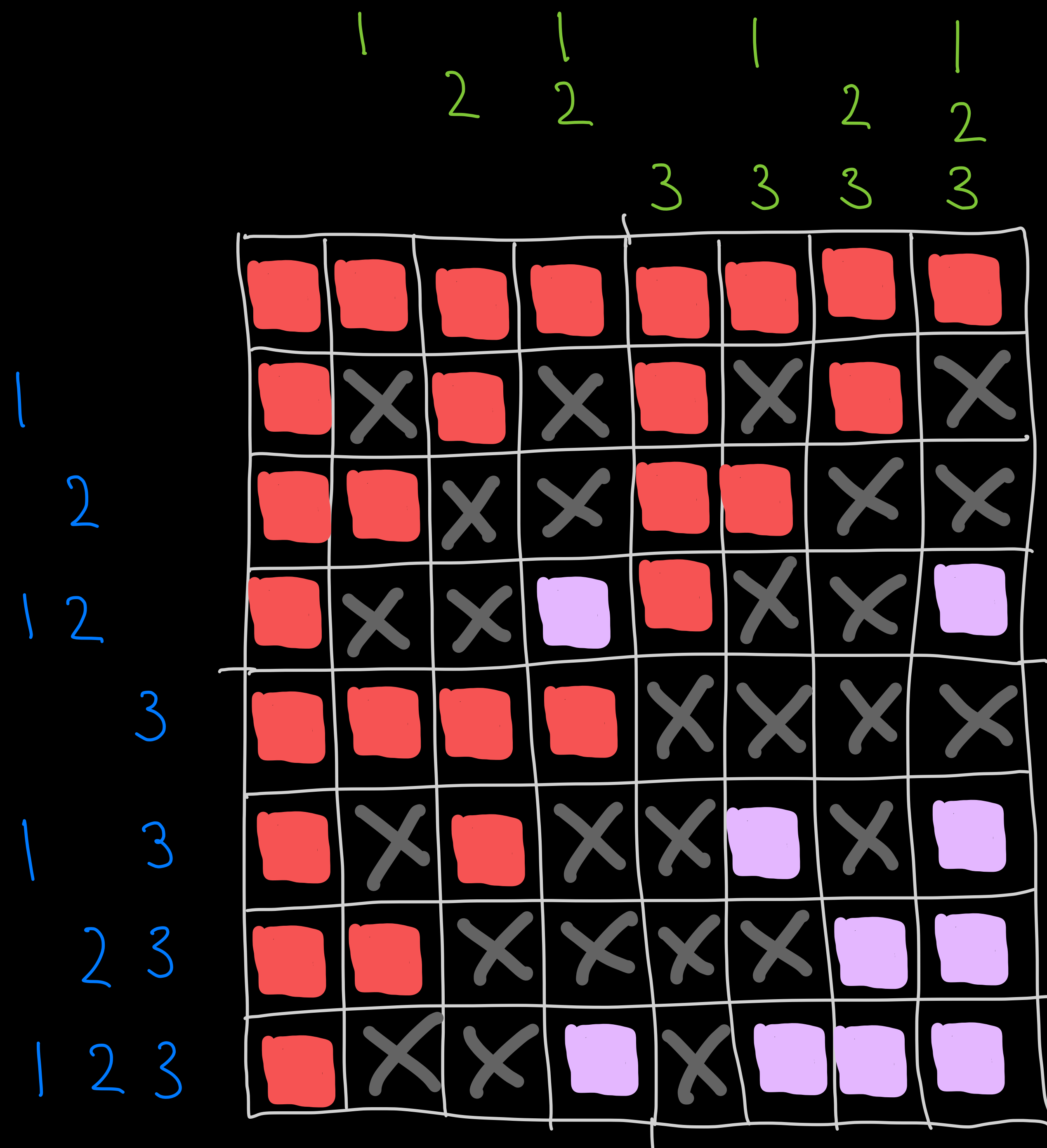
AND



TO SHOW:

AT LEAST $\left(\frac{3}{2}\right)^n$ X-FREE RECTANGLES NECESSARY TO COVER ALL ■ AND ■

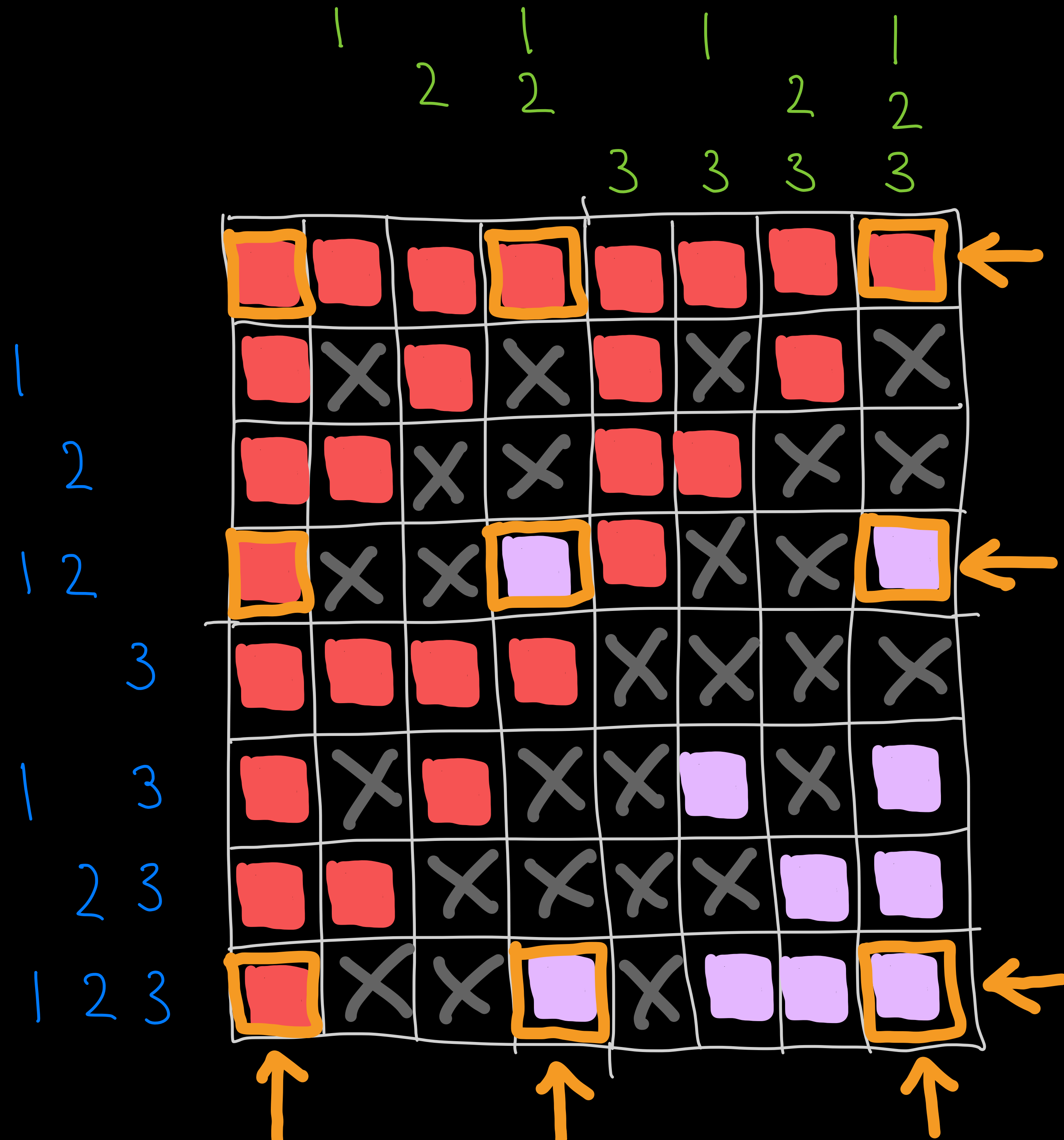
3^n



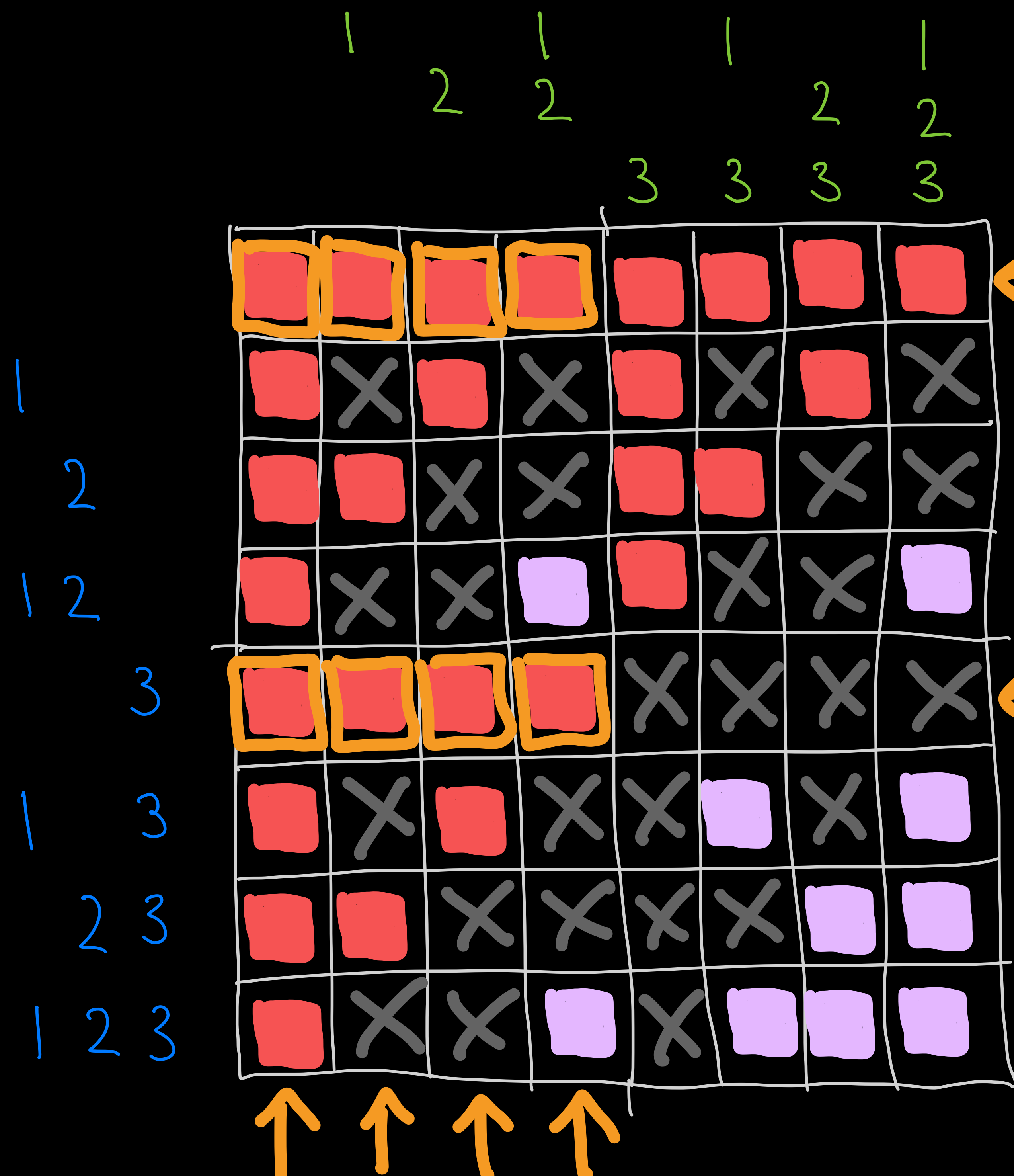
<u>TO SHOW:</u>

X - FREE RECTANGLES HAVE AT MOST 2^n ■'s

TO SHOW:

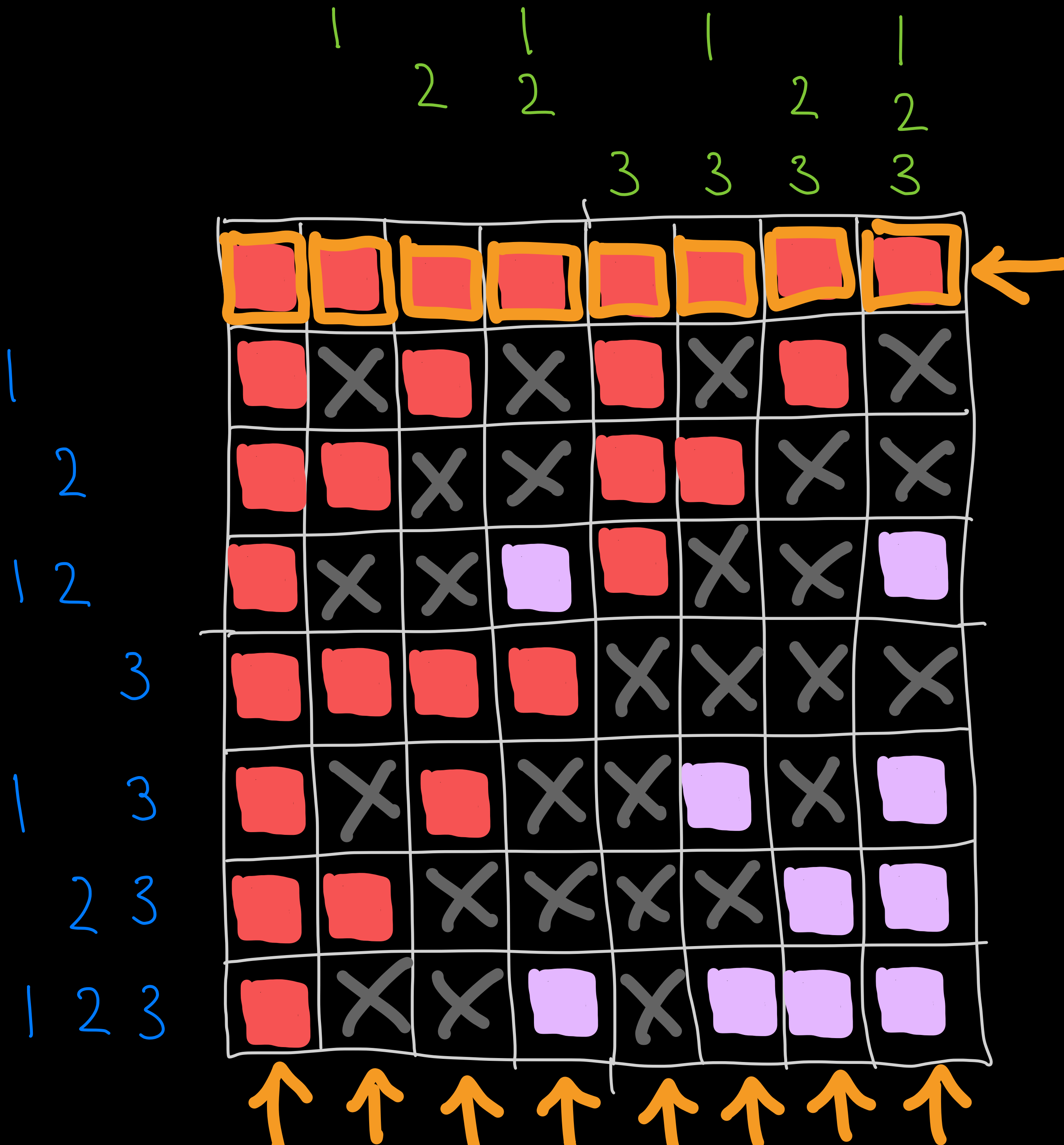


X - FREE RECTANGLES HAVE AT MOST 2^n 's



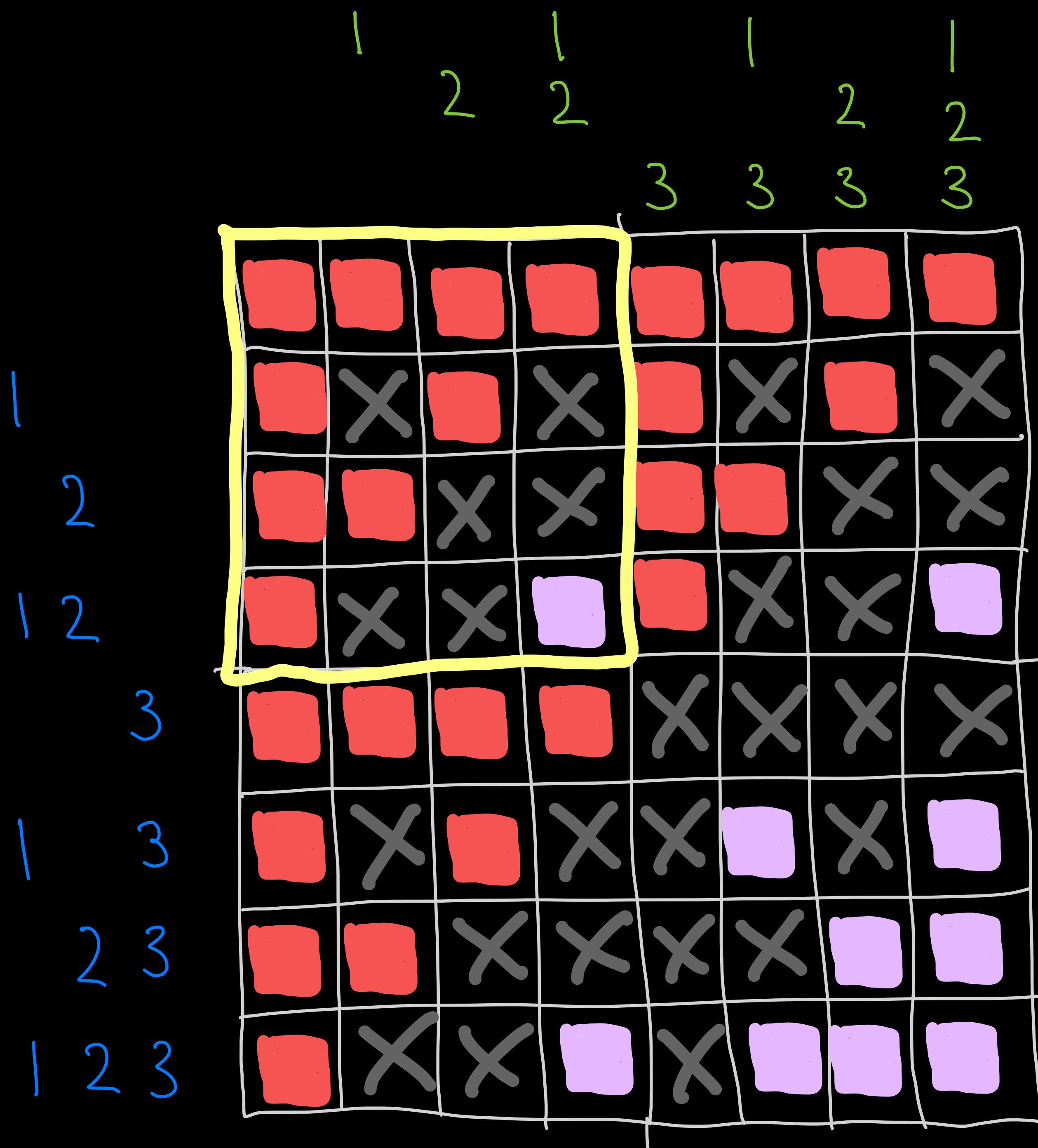
TO SHOW:

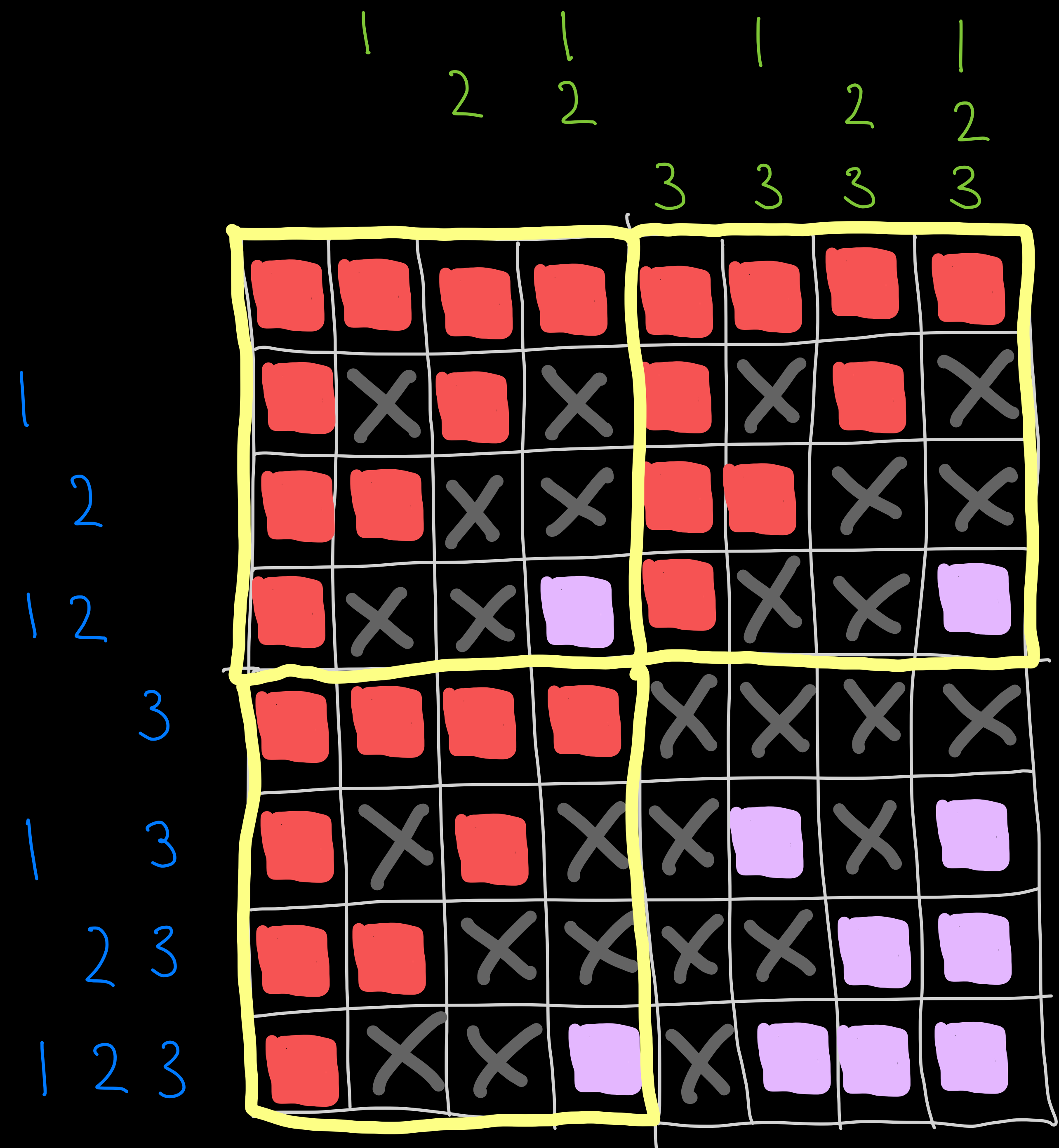
X-FREE RECTANGLES HAVE AT MOST 2^n ■'s

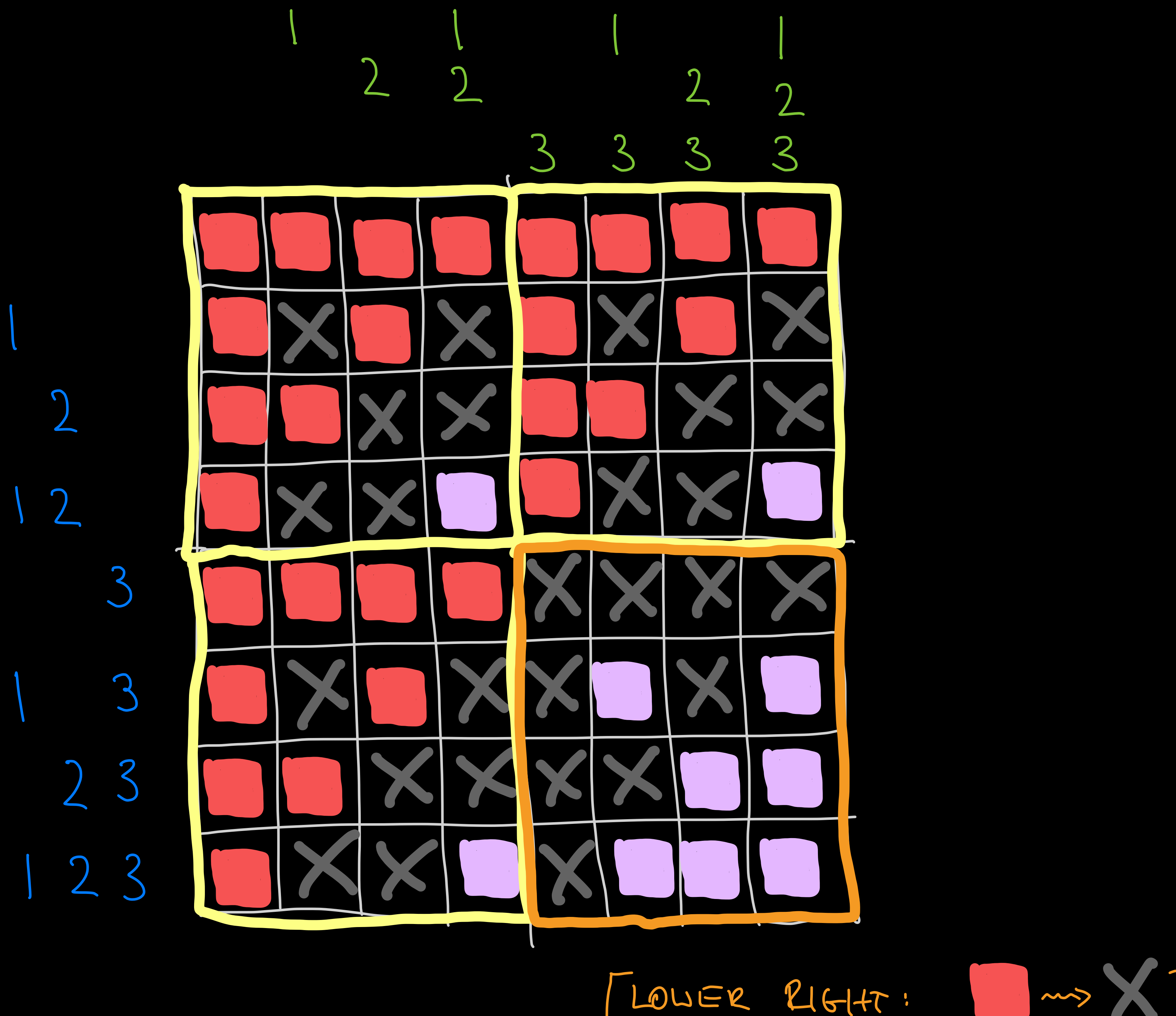


TO SHOW:

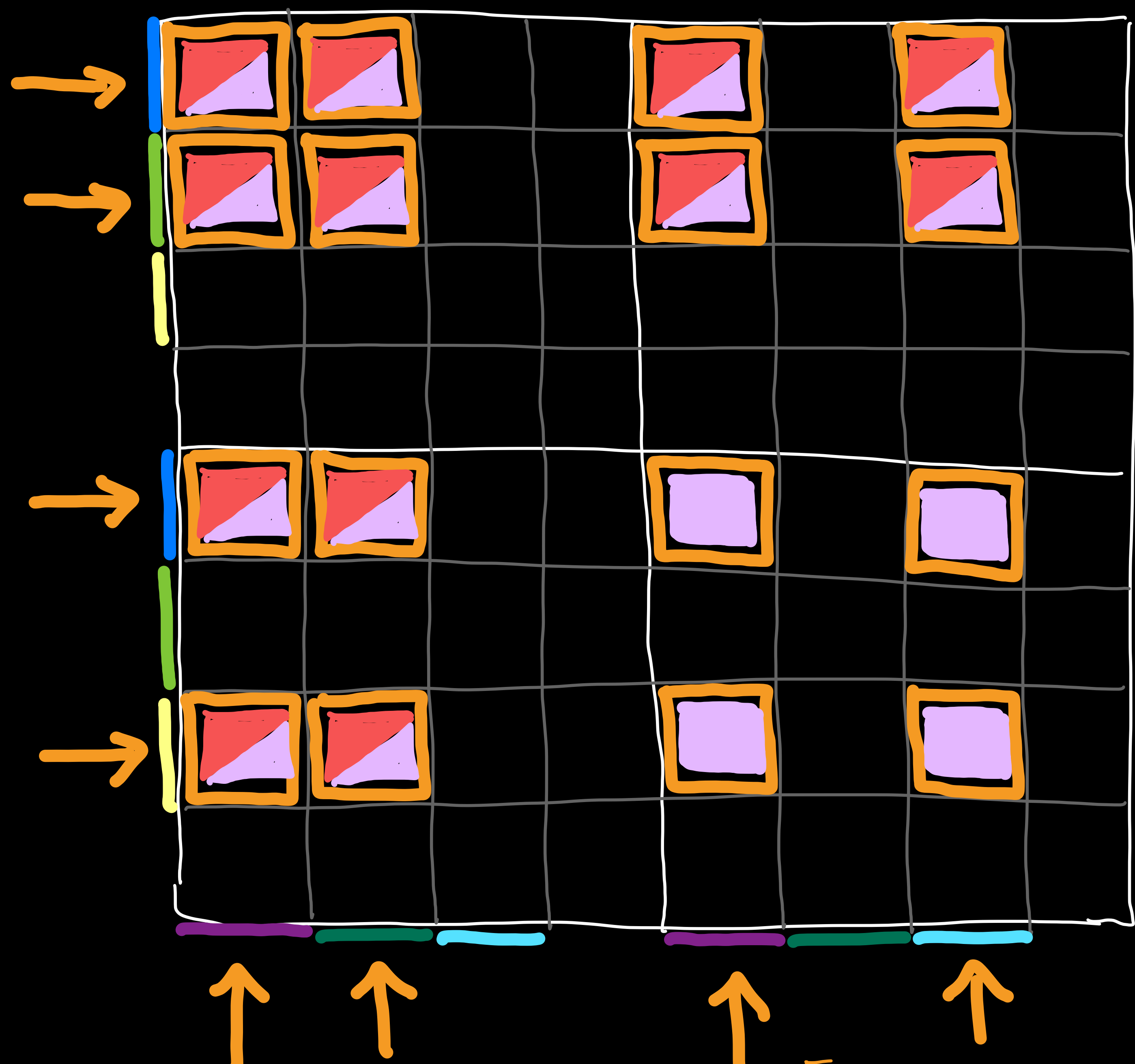
X - FREE RECTANGLES HAVE AT MOST 2^n 's


1
2
1 2
3
1 3
2 3
1 2 3
1
2
1 2
3
1 3
2 3
1 2 3


1
2
1 2
3
1 3
2 3
1 2 3
1
2
1 2
3
1 3
2 3
1 2 3

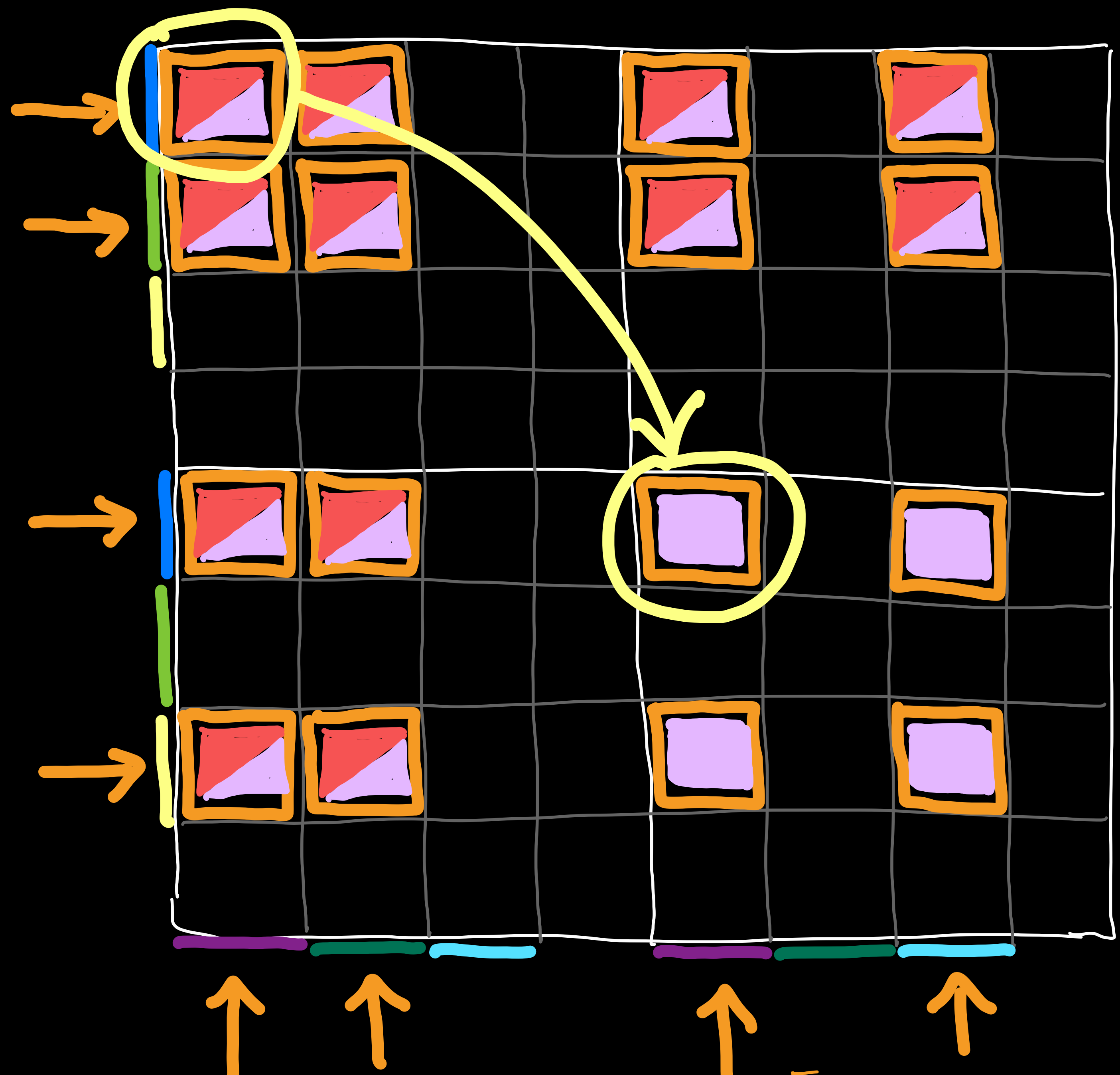

1
2
1 2
3
1 3
2 3
1 2 3
1
2
1 2
3
1 3
2 3
1 2 3
[LOWER RIGHT:]

TO SHOW : X-FREE RECTANGLES HAVE ≤ 2^n ■'s



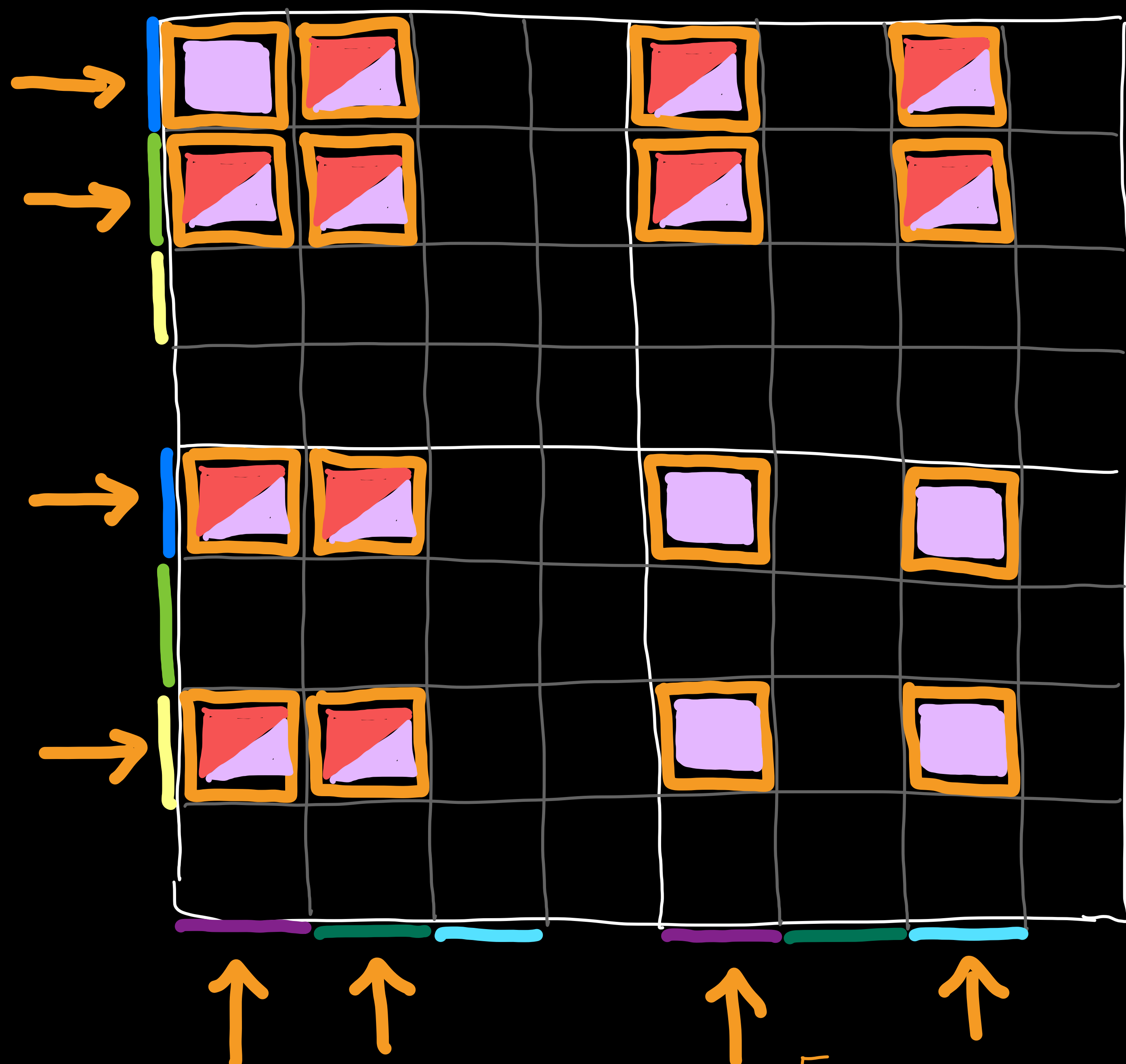
[LOWER RIGHT : ■ ⇝ X]

TO SHOW: X-FREE RECTANGLES HAVE $\leq 2^n$'s



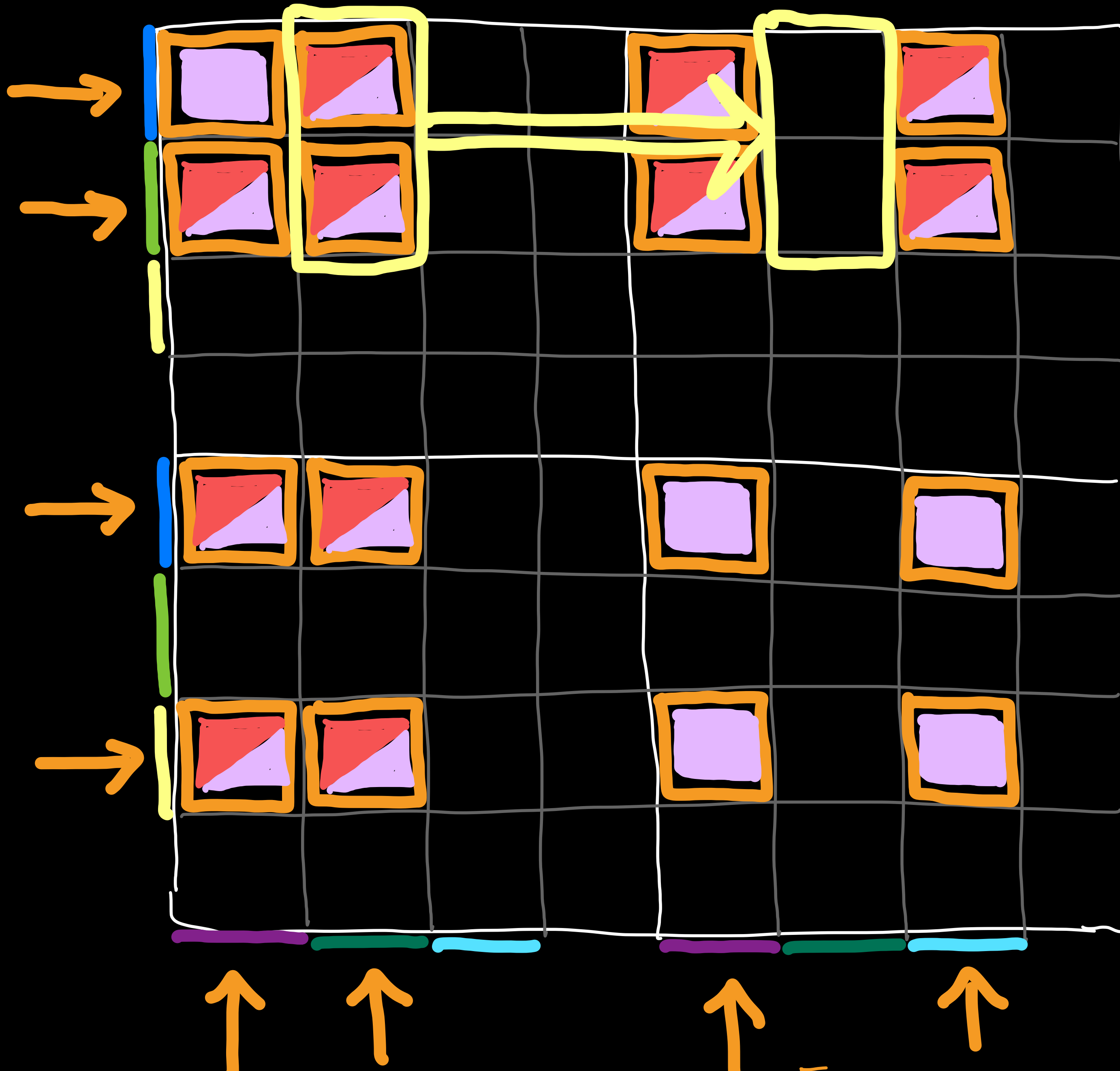
[LOWER RIGHT: $\rightsquigarrow$ X]

TO SHOW: X-FREE RECTANGLES HAVE $\leq 2^n$ ■'s



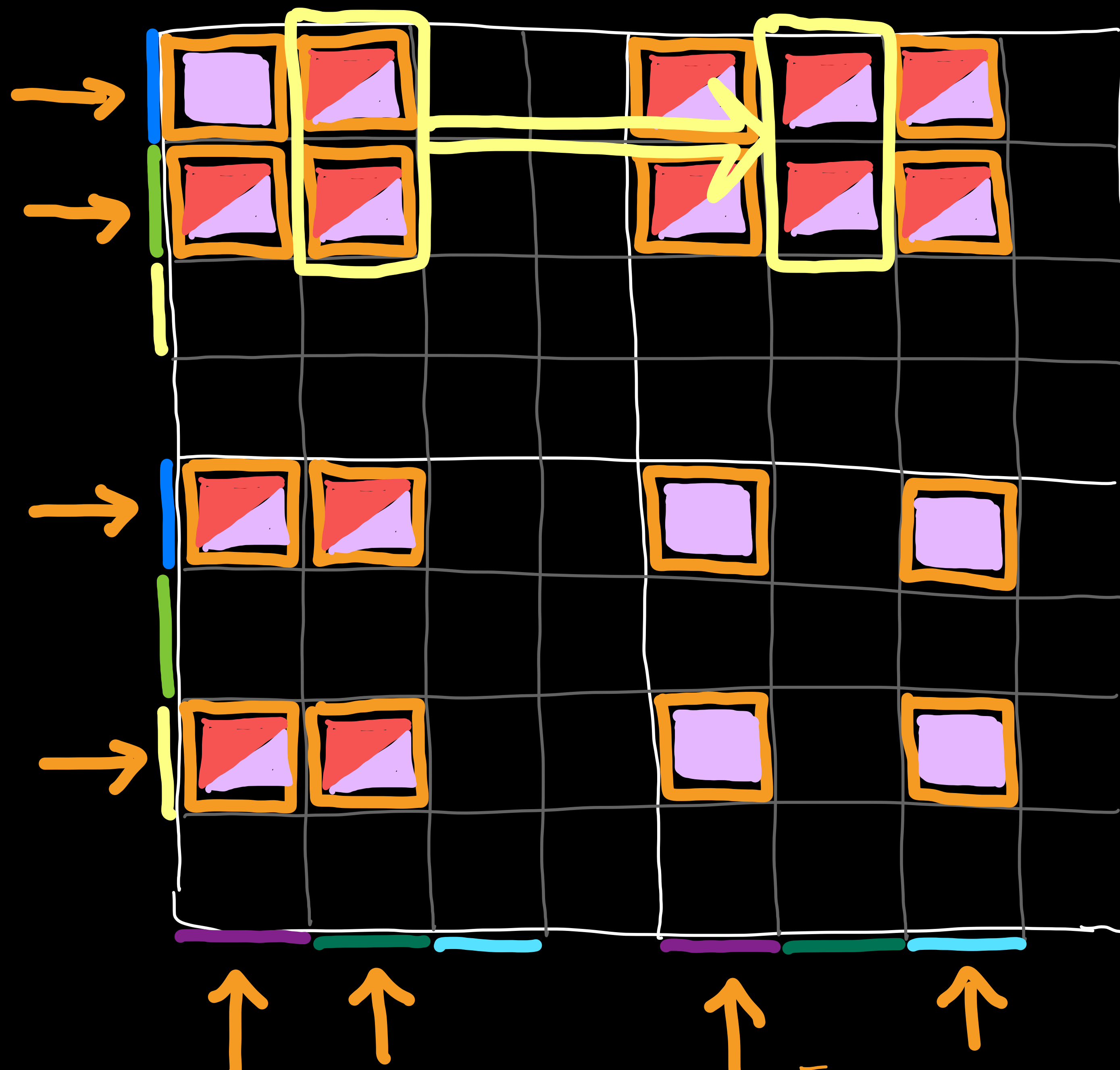
[LOWER RIGHT: ■ $\rightsquigarrow$ X]

TO SHOW: X-FREE RECTANGLES HAVE $\leq 2^n$ ■'s



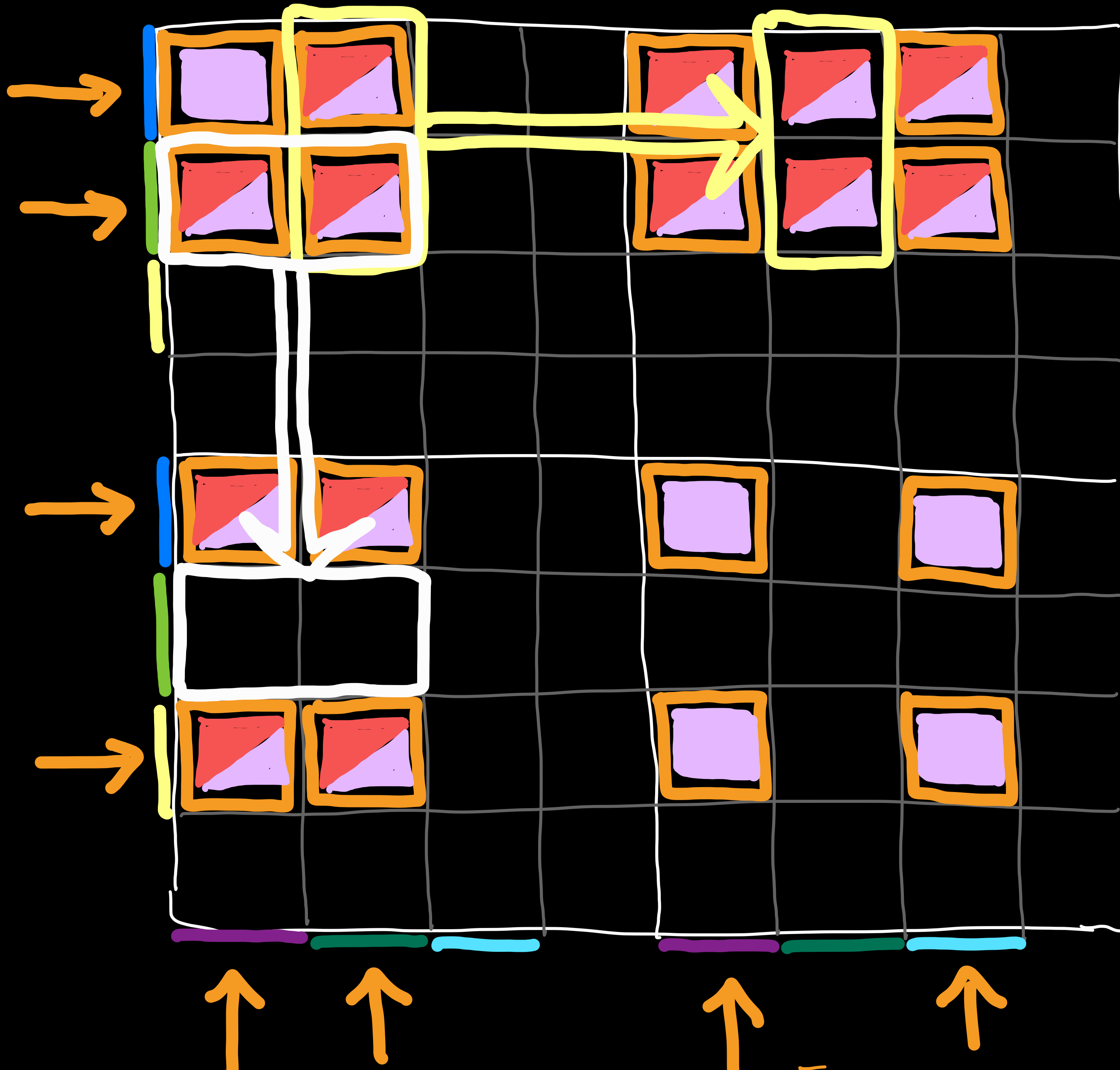
[LOWER RIGHT: ■ $\rightsquigarrow$ X]

TO SHOW: X-FREE RECTANGLES HAVE $\leq 2^n$ ■'s



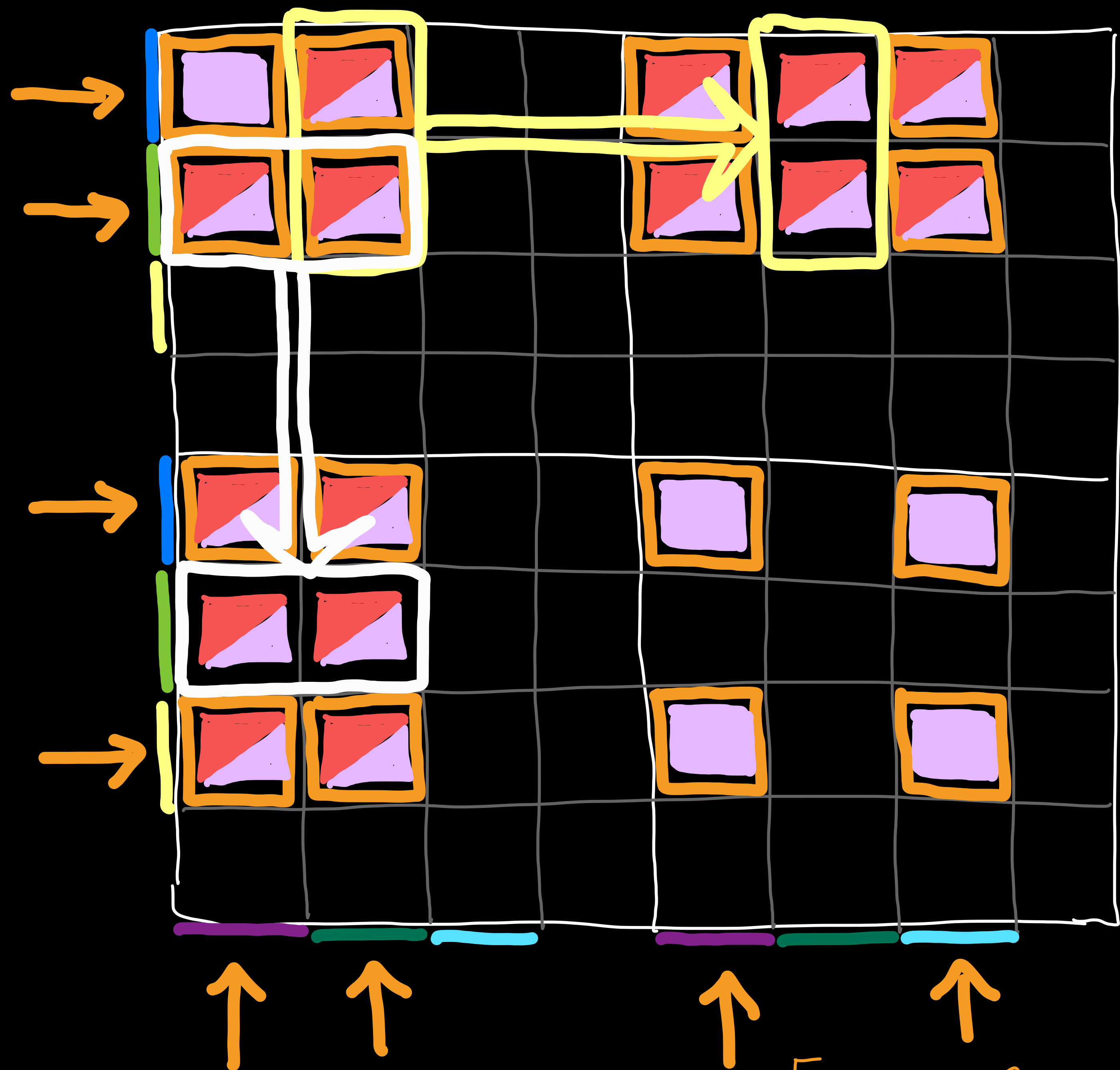
[LOWER RIGHT: ■ ⇝ X]

TO SHOW: X-FREE RECTANGLES HAVE ≤ 2^n ■'s



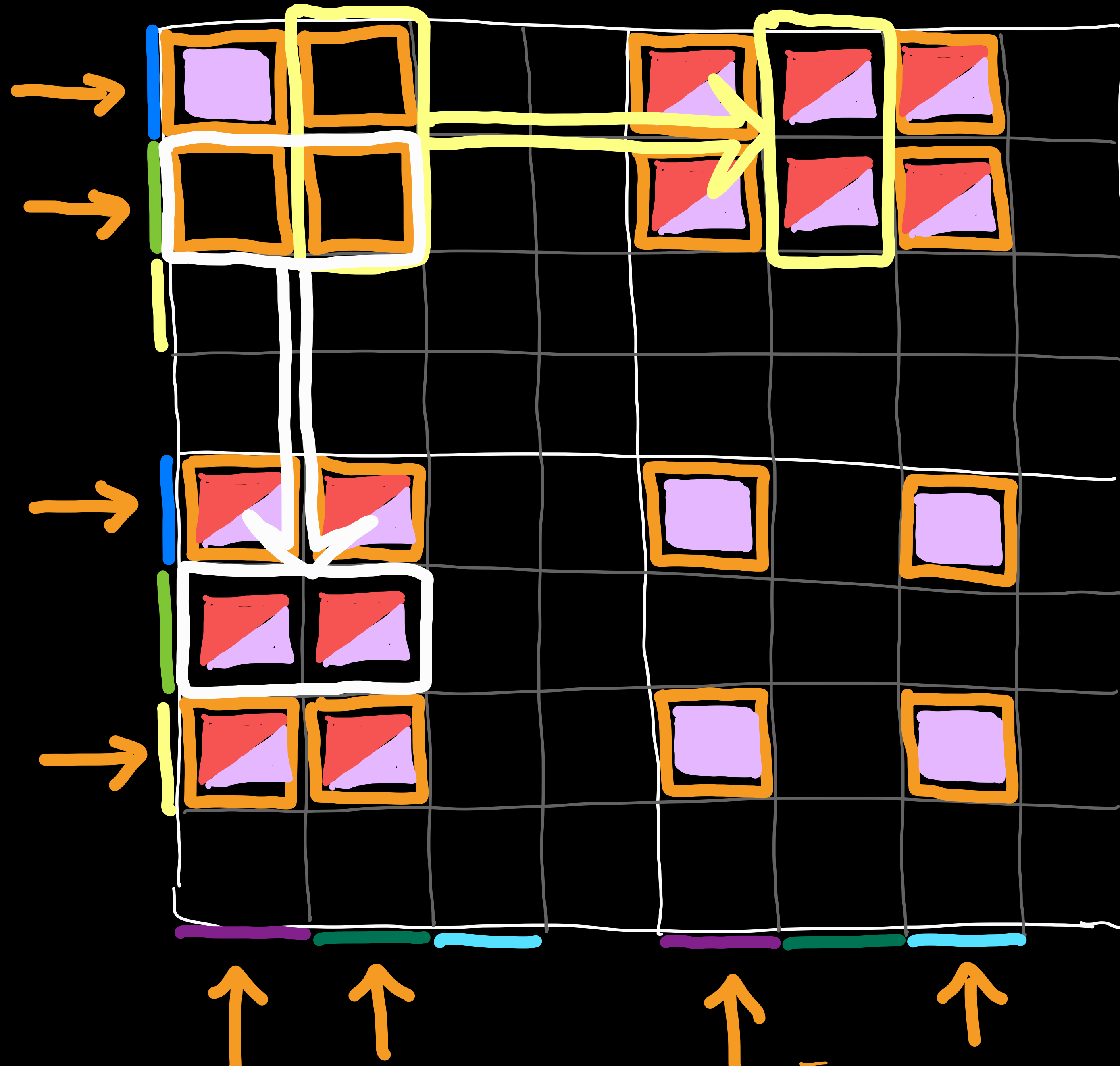
[LOWER RIGHT: ■ ⇝ X]

TO SHOW : X-FREE RECTANGLES HAVE $\leq 2^n$ ■'s



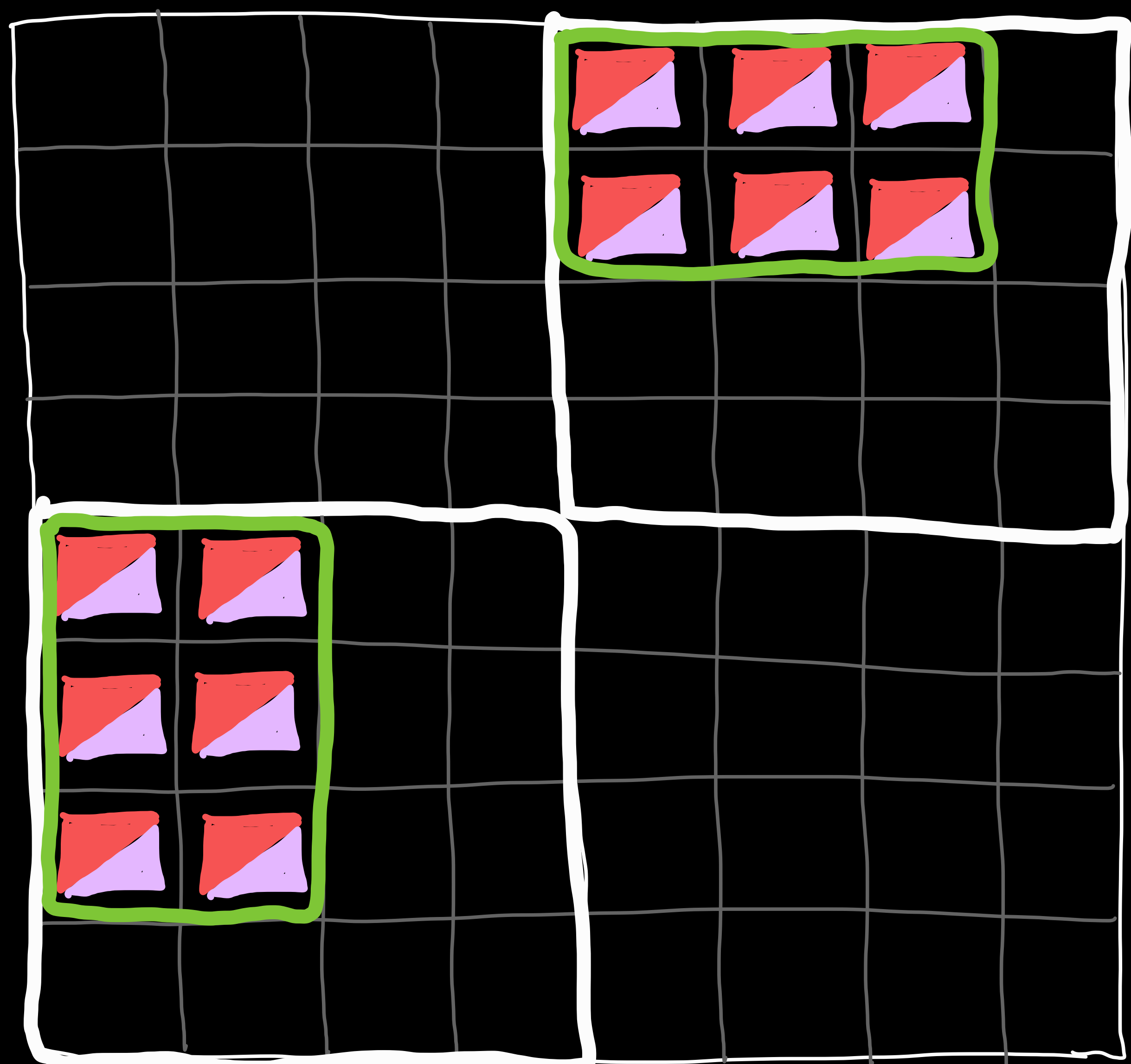
[LOWER RIGHT : ■ ⇝ X]

TO SHOW: X-FREE RECTANGLES HAVE $\leq 2^n$'s



[LOWER RIGHT: ~> X]

TO SHOW: X-FREE RECTANGLES HAVE $\leq 2^n$'s



$$XC(CORR(N)) < 2^n$$

?